Hawk05-ST Finite Enter/Page Proxies on a Prescribed Conical Spacetime

Complete complements, destructive records, conserved sectors, and conditional extrapolation

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Abstract

We assemble a self-contained finite-dimensional Enter/Page framework on a separately prescribed conical spacetime, retaining the distinction between coherent quantum return and destructive classical readout. The operational, random-state, and extrapolation constructions are presented in one compatible notation, with complete complementary outputs and explicit reference-preserving error metrics. A classical-only accessible output has sharp whole-code half-diamond recovery error at least $1 - 1/d$, independently of record length, classical heat labels, and finite adaptive processing. For nested coherent tensor release we derive the exact Page–Sen mean and, for destructive release of the same cells, a different exact cumulative-record mean. We also derive their conserved-additive-sector versions, give a simultaneous all-prefix capacity approximation for one sampled unitary, and separate these fixed-input entropy statements from whole-code recovery by exact counterexamples and a quantitative subcode-decoupling certificate. A capacity-compatibility theorem shows why an entropy-production ratio different from one cannot be introduced into a fixed tensor-release model merely by changing its clock. A rounded inverse-square-clock example and complete-channel comparison bounds make the resulting two-curve extrapolation explicit without identifying it with gravitational evaporation. The principal contribution is the error-controlled integration of these results with calibrated destructive clocks, a declared free-field packet interface, separate thermodynamic accounting, and a same-model endpoint checklist. This may be useful for distinguishing what entropy measurements, geometric reports, and receiver certificates actually establish. No priority is claimed for the standard error-correction, entanglement-breaking, Page–Sen, or thermal principles. The cone, its extension and holonomy, field data, Haar hypothesis, emission schedule, and any coherent receiver are declared inputs. Five gravitational obligations remain open. No gravitational Page curve, Einstein dynamics, spacetime emergence, or inherited characteristic quantum coefficient is constructed.

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Introduction, contribution, and evidential scope

The four separations

The Philosophy of Enter distinguishes destructive readout from quantum recovery, a classical record from a quantum-return decoder, an abstract channel from a thermal implementation, and an analogy from a physical

identification [CheungPhil]. Cheung's Enter article and Manuscript I give a finite operational characterization within a specified strategy class [CheungEnter,CheungI]. Here ordinary finite-dimensional Born instrument semantics are adopted explicitly. Their characterization is not rederived from a shortened list of requirements.

A runtime quantum name specifies access to a quantum factor. Destructive Enter consumes the measured name at the returned interface; it does not assert physical annihilation of every implementing degree of freedom. A decoder instead acts on a declared accessible output and returns a quantum target. For whole-code recovery, the target is the identity channel on an unknown code, including correlations with an untouched reference.

The spacetime construction I-B adds locations, accessible algebras, controls, resources, and calibrated finite clocks to a prescribed continuum geometry [CheungIB]. I-C makes the shredder language precise: useful classical records can survive a destructive cut without constituting a universal quantum-return channel [CheungIC]. These distinctions organize the present paper.

What is consolidated

The source records Hawk05-A, Hawk05-B, and Hawk05-C respectively concern an operational certificate, a finite Haar/Page proxy, and conditional capacity-clock extrapolation [HawkA,HawkB,CheungHawkC]. The present manuscript consolidates their finite mathematical targets rather than replacing their registered statements.

The consolidation has three parts:

• a complete operational interface on the I-B host, including the clock instrument, sharp classical obstruction, hybrid recovery, readout-and-drop, endpoint underdetermination, and corrected single-excitation histories;

• one nested coherent Haar model, its destructive counterpart, exact entropy means, whole-code comparisons, and conserved-sector calculation;

• compatible capacity clocks, conditional entropy-transfer and recovery allowances, and independent energy, reset, leakage, and geometry ledgers.

The finite results needed for this assembly are proved below. The Page–Sen formula is evaluated in Appendix . The Haar fourth-moment calculation is given in Appendix . Stinespring continuity and finite-dimensional entropy continuity are the explicitly identified external approximation inputs [KSW,Audenaert].

Source and authorship discipline

The registered Cheung manuscripts are cited by author, identifier, and supplied version when available. The supplied materials contain inconsistent authorship descriptions for Hawk05-A and Hawk05-B; those two bibliography entries are therefore source-record entries, not inferred author assignments. No authorship of this new manuscript is inferred from a cited source.

An unavailable passage is not reconstructed from its theorem label. Some component references below are documentary comparisons supplied at excerpt or summary level. In particular, no unprovided field-acquisition theorem, physical receiver, or interacting evaporation channel is imported through a citation.

The OpenAI catalogue and the Cheung comparison manuscripts bearing Oai identifiers are distinct collections. No catalogue family is a theorem premise or independently verified by this paper.

Corrections that are part of the formulation

toprule Potential ambiguity Correct formulation used here

midrule A classical lower bound holds on every individual input The bound is a channel worst-case statement, witnessed by a Bell input. Known orthogonal inputs may be reconstructed perfectly.

Residual leakage equals residual recovery error They are different optimizations; their flagged formulas have different weights.

A discrete schedule must reach its entropy peak It can skip the midpoint or the entire intermediate regime.

A snapshot is a cumulative record The complete emission-time history has its own probability law and Shannon entropy.

Page averaging forgets the sampled unitary The entropy average is over a family of fixed devices. Averaging the channel while discarding the unitary is a different task.

More postprocessing improves recovery Additional access helps when the earlier channel is a postprocessing of the later channel. Degrading an output has the opposite direction.

Holonomy supplies an additive-charge ensemble A boundary holonomy and an additive conserved charge are distinct inputs; neither derives Haar typicality.

A Maxwell potential with ghost is already a physical photon register A positive physical packet space and finite encoding must be declared separately.

An inverse-square luminosity changes the cone The luminosity is a separate supplied envelope; the host metric does not evolve.

bottomrule

The prescribed host and the field interface

Geometry, completion, and the missing apex

Fix $0 < \alpha < 1$, $c > 0$, and $\mu = 2\pi(1 - \alpha)$. The regular host is

$$M = \mathbb{R}_T \times (0,\infty)_r \times (\mathbb{R}/2\pi\mathbb{Z})_\varphi \times \mathbb{R}_z, \qquad g = -c^2\,\mathrm{d}T^2 + \mathrm{d}r^2 + \alpha^2 r^2\,\mathrm{d}\varphi^2 + \mathrm{d}z^2.$$

Its completion adds

$$W = \mathbb{R}_T \times \{o\} \times \mathbb{R}_z.$$

Thus the defect is a worldsheet, not a worldline. The time orientation is chosen so that ∂_T is future directed. Static observers have $d\tau = dT$.

On a sufficiently small angular interval,

$$(T, r, \varphi, z) \mathrm{longmapsto} (T, r\cos(\alpha\varphi), r\sin(\alpha\varphi), z)$$

is a local Minkowski isometry. It does not remove the global conical gluing. Geometry, deficit, time orientation, and clock calibration are inputs, as in I-B [CheungIB].

Proposition. The regular spacetime M in is not globally hyperbolic.

Proof. Choose $r_{*} > 0$, fixed φ_{*}, z_{*}, and a time $a > r_{*}/c$. The causal diamond between

$$p = (-a, r_*, \varphi_*, z_*), \qquad q = (a, r_*, \varphi_*, z_*)$$

contains

$$p_j = (0, r_j, \varphi_*, z_*), \qquad r_j \mathrm{downarrow}\, 0.$$

Radial paths have length $r_{*} - r_j < ca$. The sequence has no convergent subsequence in M, since its only completion limit lies on the omitted worldsheet. The diamond is therefore not compact.

This elementary argument agrees with I-B-G [CheungIBG]. It prevents use of a globally-hyperbolic Green theorem without checking its hypotheses.

Declared scalar and Maxwell data

For field formulas in this subsection use $t = cT$, up to a fixed origin, so the metric is $-dt^2 + h$. Fix a flat complex line bundle with angular holonomy $e^{2\pi i\beta}$, $0 \le \beta < 1$, and a nonnegative mass m_f. Let $A_{F,\beta}$ be the declared nonnegative Friedrichs spatial realization of the scalar operator. Its spectral wave propagator is

$$S_F(t) = \frac{\sin(t\sqrt{A_{F,\beta}})}{\sqrt{A_{F,\beta}}}, \qquad S_F(t)|_{\lambda=0} = t.$$

The retarded and advanced operators are

$$G_F^{+}(t) = 1_{\{t \ge 0\}} S_F(t), \qquad G_F^{-}(t) = -1_{\{t \le 0\}} S_F(t).$$

Assumption. Any field interpretation supplies:

• the field representation, bundle, mass, extension, and source and test domains;

• the Green operator on those domains, including the applicable finite-propagation statement;

• a positive state or covariance and the renormalization conventions for any composite observable used;

• a positive physical packet space and a specified finite encoding or truncation;

• a separate acquisition or receiver channel if an unknown continuum input is to be converted into the finite model. Only the finite encoding and its declared matrices enter the finite proofs below.

The scalar kernel and its extension-dependent domain are documented in I-B-G [CheungIBG]; the geometric and diffracted pieces are not silently substituted into a different extension or test space. No compact-spacetime exactness theorem is inferred from an exact sequence on a spectral domain.

For a photonic interpretation, I-B-G-S records the source-tested Maxwell–Friedrichs identity with a compatible scalar ghost,

$$C_1 G^{\pm}_{(1)} = G^{\pm}_{(0)} C_1, \qquad \nabla^a_x W^{(1)}_{ab}(x, x') = -\nabla_{b'} W^{(0)}(x, x').$$

These are documentary field inputs, not locally re-proved Maxwell theorems [CheungIBGS]. A finite photon encoding must still satisfy Assumption ; an indefinite potential/ghost description is not itself a positive quantum-register Hilbert space.

The componentwise tensor domain and the positive tensor-core selection discussed in I-B-G-S-B are separate from this Maxwell interface [CheungTensor]. They are not relabelled photons or physical gravitational Hessians.

Vacuum polarization, Wick squares, stress tensors, and the holonomy-odd current are used only as prescribed-background data from the I-B-VP records [CheungVP,CheungVP0]. A frozen current and a Maxwell solution driven by that current are distinct objects; I-B-VP-4 is the separate frozen-source response comparison [CheungVP4]. None of these data supplies interacting evaporation or backreaction.

Finite packets do not constitute continuum acquisition

Let $\mathcal{K}_adm$ be a declared finite-dimensional packet code in a positive field representation, and let

$$E_{\mathrm{pkt}} : \mathcal{K}_{\mathrm{adm}} \to H_{\mathrm{int}}$$

be a specified isometry. A finite channel acts on $\mathscr{B}(H_int)$. It has no action on an arbitrary field state until a field-to-register channel is supplied.

This typing distinction is the non-acquisition boundary in I-B-ECHO and in the supplied X-J-ST-2 comparison [CheungEcho,CheungXJ2]. It is not a universal impossibility theorem for all detectors. The source-field model of Hawk02-C is a useful separate packet-channel comparison [CheungHawk02C]. Em03-A and Em03-B distinguish a fixed-background emission law from an apparatus-model packet readout [CheungEm03A,CheungEm03B]. Their horizons and states are not identified with the cone apex.

Proposition. A conserved symmetric traceless tensor need not source its background through an Einstein equation.

Proof. In a flat four-dimensional chart with signature $(-+++)$, let $e > 0$ and

$$t_{\mu\nu} = \mathrm{diag}\,(3e, e, e, e).$$

It is constant, symmetric, divergence free, and traceless. For zero cosmological constant the Einstein tensor of that chart vanishes, whereas $\kappa t_\{\mu\nu\} \neq 0$ for $\kappa \neq 0$.

Thus even a correctly normalized field stress does not become a metric-update rule merely by being placed beside . This is the source separation retained in I-B [CheungIB].

Operational conversion: clocks, access, and the shredder

Complete interfaces and calibrated destructive clocks

Complete outputs and metrics

All code and register spaces in the finite results are finite dimensional. Let $C \simeq \mathbb{C}\wedge d$, $d \geq 2$, be the entire unknown code. An external reference Q is untouched and is not available to the decoder.

Definition. A complete interface is an isometry

$$V : C \to R \otimes B.$$

Its accessible and complete complementary channels are

$$\mathcal{M}(X) = \mathrm{Tr}_B\,(VXV^\dagger), \qquad \mathcal{C}(X) = \mathrm{Tr}_R\,(VXV^\dagger).$$

Every unreturned preparation purifier, measurement environment, lost mode, controller output, and other admitted implementation factor belongs to *B*. A mixed fixed ancilla is represented by a fixed purification. An input-dependent preparation belongs to the encoding, not to the fixed-ancilla ledger.

For a state τ, write

$$\mathcal{T}_\tau(X) = \mathrm{Tr}\,(X)\tau.$$

Use

$$d_{\mathrm{tr}}(\rho,\sigma) = \tfrac{1}{2}\|\rho-\sigma\|_1, \qquad d_\diamond(\Phi,\Psi) = \tfrac{1}{2}\|\Phi-\Psi\|_\diamond,$$
$$\varepsilon_{\mathrm{rec}}(\mathcal{N}) = \inf_{\mathcal{D}\,:\,R\,\to\,C\ \mathrm{CPTP}} d_\diamond(\mathcal{D}\mathcal{N},\mathrm{id}_C), \quad \delta_{\mathrm{full}}(\mathcal{C}) = \inf_{\tau\,\in\,\mathcal{S}(B)} d_\diamond(\mathcal{C},\mathcal{T}_\tau).$$

The diamond norm includes arbitrary references [Watrous]. Define

$$a_d = 1 - d^{-2}, \qquad b_d = 1 - d^{-1}.$$

The infima in are attained by compactness of finite-dimensional channel and state sets.

A decoder is deterministic. It may use input-independent fresh ancillas and randomness, but no excluded input-bearing quantum factor or unlisted preshared entanglement. Supplying such a factor changes the interface.

Lemma. CPTP pre- and postprocessing contract *d_{◇}*. Consequently

$$\left|\varepsilon_{\mathrm{rec}}(\mathcal{N}) - \varepsilon_{\mathrm{rec}}(\mathcal{N}')\right| \mathrm{led}_\diamond\,(\mathcal{N},\mathcal{N}').$$

For compatible complete stage channels,

$$d_\diamond(\widetilde{F}_n\cdots\widetilde{F}_1, F_n\cdots F_1) \le \min\left\{1, \sum_{j\,=\,1}^{n} d_\diamond(\widetilde{F}_j, F_j)\right\}.$$

Proof. Trace distance contracts on every referenced input. Taking suprema proves channel contraction. Insert any decoder into the triangle inequality and take infima in both directions for recovery stability. For , subtract the compositions and replace one factor at a time. Each difference is surrounded by channels, so contraction and the triangle inequality apply.

The complete intermediate types in include histories, failures, controls, and surviving factors. A logical-only estimate cannot price an omitted arrival or failure flag. The corresponding successive-clock discipline is documented in X-K-ST [CheungXK].

The I-B clock instrument

Let *K* ≃ ℂ^*D_clk* be a clock, with orthonormal basis *{|τ_j⟩}_{j = 0}^{D_clk − 1}*, and let *A* contain all surviving finite quantum registers. The calibration supplies finite codes *j*, optional words *w_j*, and geometric labels *t_j*. Put

$$D_j = \langle\tau_j|_K \otimes I_A.$$

Theorem. The map

$$\mathcal{M}_K(X) = \sum_j |j, w_j\rangle\langle j, w_j|_Y \otimes D_j X D_j^\dagger$$

is CPTP, and

$$\mathrm{Tr}_Y \mathcal{M}_K(X) = \mathrm{Tr}_K X.$$

Every identity holds with arbitrary untouched references. On a state *ρ*, branch *j* has probability *p_j = Tr (D_jρD_j^{†})*; its normalized conditional residual is *D_jρD_j^{†}/p_j* only when *p_j > 0*. The clock name *K* is not returned.

Proof. The recorded Kraus operators are *L_j = |j, w_j⟩_Y ⊗ D_j*, and

$$\sum_j L_j^\dagger L_j = \sum_j |\tau_j\rangle\langle\tau_j|_K \otimes I_A = I_{KA}.$$

Expand *X = ∑_{j, ℓ} |τ_j⟩⟨τ_{ℓ}| ⊗ X_{jℓ}*. The recorded blocks are *X_jj*, whose sum is *Tr_K X*. The same

expansion allows operator-valued blocks on $A \otimes Q$. The branch statement follows from positivity and normalization.

This is the locally proved identity appearing in I-B, I-C, and En09-A [CheungIB,CheungIC,CheungEn09]. Finite failures and unused labels must be retained in the output convention. They are not deleted by postselection.

A complete readout dilation is

$$W_{\mathrm{read}}|\tau_j\rangle = |j\rangle_Y|j\rangle_L,$$

with L unreturned. The dilation preserves superpositions jointly; the returned Y-interface is classical after L is traced out.

Proposition. For distinct labels t_j:

• exact agreement of a supported outcome with the coordinate time of one fixed event forces a deterministic outcome;

• $\mathscr{M}_K$ has no channel left inverse on the entire $K \otimes A$ input space when $D_{clk} \geq 2$;

• the clock record law does not determine the residual state;

• no finite matrices T_K, H_K satisfy $[T_K, H_K] = i\hbar I_K$ for $\hbar > 0$.

Proof. For (i), if every positive-probability outcome obeys $t_j = T(e)$, distinctness permits at most one supported label; normalization makes its probability one. For (ii), the two states

$$\frac{|\tau_0\rangle + |\tau_1\rangle}{\sqrt{2}}, \qquad \frac{|\tau_0\rangle - |\tau_1\rangle}{\sqrt{2}}$$

give the same output when tensored with any fixed residual state. For (iii), the inputs $|\tau_0\rangle\langle\tau_0| \otimes \mathrm{proj}\, a$ and $|\tau_0\rangle\langle\tau_0| \otimes \mathrm{proj}\, b$, with a, b orthogonal, have the same deterministic record and different residuals. For (iv), take the trace of the proposed commutator identity.

Nontrivial stochastic clock labels may instead refer to a supplied history-event map $j \mapsto e_j$, with $T(e_j) = t_j$. They need not equal the coordinate of the actual readout event. A general clock-basis unitary changes the observable, not the spacetime coordinates. If it merely relabels a nondegenerate pointer, it must permute its eigenspaces up to phases [CheungIB].

History conditioning with a consumed clock

For fixed unitaries U_j on a matter system A, define

$$J_{\mathrm{clk}}|\psi\rangle = D_{\mathrm{clk}}^{-1/2} \sum_j |\tau_j\rangle \otimes U_j|\psi\rangle.$$

Proposition. The map J_{clk} is isometric, and

$$\mathscr{M}_K(J_{\mathrm{clk}} X J_{\mathrm{clk}}^\dagger) = \frac{1}{D_{\mathrm{clk}}} \sum_j \mathrm{proj}\, j_Y \otimes U_j X U_j^\dagger.$$

Record-controlled application of $U_j^\dagger$ recovers the encoded matter input, including references. It does not invert the clock instrument on its full input space.

Proof. Orthogonality gives $J_{clk}^\dagger J_{clk} = I_A$, and $D_j J_{clk} = D_{clk}^{-1/2} U_j$. Substitute into . The controlled inverse then returns X in every branch.

Taking $U_j = e^{-itjH_A/\hbar}$ gives a finite history encoding. The stronger Page–Wootters interpretation requires its stationarity and clock-covariance assumptions; these are not consequences of conditioning alone [PageWootters,CheungEn09]. A reference-extended dynamical statement requires stationarity of the complete extended preparation, not merely of its clock–matter marginal.

Scheduled containment and access-preserving host placement

Locality with all controls included

A finite execution uses actual quantum ports, controller memories, guards, records, and resource wires. A control is either preinstalled task data or an explicit causal record wire, as required by the typed observation-boundary discipline [CheungEn03,CheungXK].

At each tick let the complete channel be a product of block channels over a supplied partition. Starting from terminal sites *Y*, define backward ancestors by adjoining every input block meeting the current backward set.

Theorem. Every terminal observable on *Y* pulls back to an observable on its initial ancestor set *N_0(Y)*. The terminal reduction factors through the initial reduction to *N_0(Y)*. For a joint reference conclusion, the relevant input is the joint ancestor–reference reduction.

If every dependency edge has a certified future-causal path in , every scheduled ancestor lies in the prescribed metric past of the output. Actual unconditional intervention influence is contained in this scheduled relation.

Proof. A block disjoint from the backward support sees the identity. Its channel adjoint is unital and preserves that identity. A meeting block enlarges support only to its own inputs. Backward induction proves the inclusion. Testing all terminal observables gives the reduced-channel factorization; tensoring with the reference algebra gives the joint version. Concatenating the supplied causal edge paths proves metric containment. An intervention outside the ancestor network acts only on an identity in the adjoint calculation and changes no terminal expectation.

Scheduled ancestry need not equal actual influence: a formally two-site block can implement the identity. Nor is it the entire metric causal past. This is a containment certificate on a prescribed host, not a derivation of a Lorentz cone [CheungIB,CheungVIIJ]. A small implementation error does not prove exact causal containment for an erroneous device.

Target-adapted null transport

Choose a convex developing spatial patch with chart $\chi: U \to \mathbb{R}^3$, two distinct points $x_0, x_m \in U$, and

$$L = \|\chi(x_m) - \chi(x_0)\|_2.$$

For an integer $m \geq 1$, install sites

$$\chi(x_j) = \chi(x_0) + \frac{j}{m}(\chi(x_m) - \chi(x_0)), \qquad \Delta T = \frac{L}{mc}.$$

Place equal-dimensional registers $A_0, \ldots, A_m$ at these sites.

Proposition. Prepare $A_1, \ldots, A_m$ in independent fixed blanks and apply $SWAP_{A_{j-1}A_j}$ at tick j. The destination receives the input of A_0 exactly, including all reference correlations, at time $T_d + L/c$. All other route registers are in a jointly input-independent blank state.

Proof. After tick j, the payload–reference state is in A_jQ, and all other route registers are blank and independent. SWAP proves this invariant by induction. Each hop has distance $L/m = c\Delta T$, so its tick-boundary dependencies admit null developed paths. The total elapsed time is $m\Delta T = L/c$.

The protocol uses m SWAPs, $m + 1$ payload-sized registers, and m ticks. With a q-qubit clock and p-qubit payload registers, the named laboratory peak is at most

$$q + p(m + 1),$$

with reference storage charged separately. This is the target-adapted contract of I-B, not exact attainment on an arbitrary fixed lattice [CheungIB,CheungEn08B]. No within-tick Hamiltonian implementing a spatial SWAP has been derived.

What a host conversion preserves

Proposition. Let $W_R: R \to R$ and $W_B: B \to B$ be isometries, and set

$$\widehat{V} = (W_R \otimes W_B)V.$$

With the corresponding accessible algebras and off-support CPTP compressions,

$$\varepsilon_{\mathrm{rec}}(\widehat{\mathcal{N}}) = \varepsilon_{\mathrm{rec}}(\mathcal{N}), \qquad \delta_{\mathrm{full}}(\widehat{\mathcal{C}}) = \delta_{\mathrm{full}}(\mathcal{C}).$$

Adding fixed pure spectators also changes neither quantity.

Proof. A CPTP compression left-inverts each isometric embedding:

$$X \longmapsto W^\dagger X W + \mathrm{Tr}\,((I - WW^\dagger)X)\omega.$$

Composing decoders with the embedding or compression gives both recovery inequalities. For leakage, embed any replacement comparison to obtain one inequality; compress an arbitrary replacement comparison on $\mathcal{B}$ to obtain the reverse inequality. Contraction and isometric trace-norm invariance finish the proof.

For classical registers the identification must preserve the declared diagonal algebra; a matrix representation does not grant access to coherent label superpositions. Proposition is the precise finite conversion: locating or encoding an already specified register does not create a field receiver, a metric, or a new recovery resource.

Recovery, the sharp shredder bound, and the metric dictionary

Global preservation and exact restricted recovery

Theorem. For a complete isometry $V\colon C \to R \otimes B$:

• the complete channel Ad_V has a CPTP left inverse and preserves every referenced operator's trace norm;

• $\mathcal{N}$ exactly recovers the whole code if and only if $\mathcal{C} = \mathcal{T}_{\tau_B}$ for one state τ_B;

• if $s = \operatorname{rank} \tau_B$, there are an isometry $T\colon C \otimes \mathbb{C}^s \to R$ and a state σ such that

$$\mathcal{N}(X) = T(X \otimes \sigma)T^\dagger.$$

An exact decoder on all of R is

$$\mathcal{D}(Y) = \operatorname{Tr}_{\mathbb{C}^s}(T^\dagger Y T) + \operatorname{Tr}((I - TT^\dagger)Y)\omega_C.$$

In particular, $\dim R \geq ds$. All statements preserve arbitrary references.

Proof. For a fixed code state ω_C, the map

$$\mathcal{L}_V(Y) = V^\dagger Y V + \operatorname{Tr}((I - VV^\dagger)Y)\omega_C$$

is CP, trace preserving, and left-inverts Ad_V. An isometry preserves all nonzero singular values, also after reference extension.

Suppose an exact decoder exists and dilate it by $W\colon R \to C \otimes G$. The composite $(W \otimes I_B)V$ dilates the identity channel. The identity has a rank-one Choi operator; positivity implies that every Kraus vector in any such dilation is proportional to its identity Kraus vector. Hence

$$(W \otimes I_B)V|\psi\rangle = |\psi\rangle_C|\chi\rangle_{GB}$$

for one fixed normalized vector χ. Tracing CG proves complement constancy.

Conversely, diagonalize the nonzero support of τ_B:

$$\tau_B = \sum_{\alpha=1}^{s} \lambda_\alpha \operatorname{proj}\alpha, \qquad \lambda_\alpha > 0.$$

Write

$$V|i\rangle = \sum_\alpha \sqrt{\lambda_\alpha}|u_{i\alpha}\rangle_R|\alpha\rangle_B.$$

The complementary matrix-unit identities $\mathcal{C}(kb\ ij) = \delta_{ij}\tau_B$ give

$$\langle u_{j\beta} \mid u_{i\alpha}\rangle = \delta_{ij}\delta_{\alpha\beta}.$$

These ds orthonormal vectors define T. Taking $\sigma = \sum_{\alpha} \lambda_{\alpha} \operatorname{proj}\alpha$ gives . The two terms of are CP, their trace effects sum to I_R, and substitution recovers every matrix unit. Linearity gives the reference statements.

This is the established exact correction structure [KnillLaflamme,Stinespring]. Comparable complete-output formulations are recorded in the Hawk04 and En07 manuscripts [CheungHawk04A,CheungHawk04B,CheungEn07A,CheungEn07B]. It is not a new error-correction principle.

Corollary. If the accessible channel is replacement, the complete complementary output exactly recovers the code. Both complementary outputs cannot independently recover the entire code for $d \geq 2$. A merely nonconstant complement does not imply that it independently recovers the code.

Exact recovery need not make the raw accessible output pure: for pure logical input, its entropy in is $S(\sigma)$.

Proof. Interchange the two output factors in Theorem . A replacement channel cannot recover two distinct inputs. The entropy assertion follows directly from .

The destination qualification is consistent with no-hiding [BraunsteinPati]; it is not a claim that every informative residual is an independent copy.

Replacement benchmarks and approximation

Let

$$|\Phi_d\rangle = \frac{1}{\sqrt{d}} \sum_{i=1}^{d} |i\rangle_Q |i\rangle_C, \qquad \Phi_d = |\Phi_d\rangle\langle\Phi_d|, \qquad J_{\mathrm{n}}(\mathscr{F}) = (\mathrm{id}_Q \otimes \mathscr{F})(\Phi_d).$$

Lemma. Every replacement accessible channel has recovery error *a_d*, and every accessible channel has recovery error at most *a_d*. Moreover,

$$\tfrac{1}{2}\|J_{\mathrm{n}}(\mathscr{F}) - J_{\mathrm{n}}(\mathscr{G})\|_1 \mathrm{led}_{\diamond}(\mathscr{F}, \mathscr{G}) \le \tfrac{d}{2}\|J_{\mathrm{n}}(\mathscr{F}) - J_{\mathrm{n}}(\mathscr{G})\|_1.$$

If *d_{◇}(𝒩, 𝒯_{τ}) ≤ η*, then

$$\varepsilon_{\mathrm{rec}}(\mathscr{N}) \ge [a_d - \eta]_+, \qquad [x]_+ = \max\{0, x\}.$$

Proof. A decoded replacement sends *Φ_d* to *I_Q/d ⊗ ω_C*, whose Bell overlap is *d^{−2}*. The Bell test gives error at least *a_d*. Preparing *I_C/d* attains it: complete depolarization is the uniform average of *d^2* Weyl conjugations, one being identity. Separating that term bounds its half-diamond distance from identity by *a_d*. This constant decoder also supplies the universal cap.

The left Choi inequality tests the Bell input. For a pure referenced input write *|ψ⟩ = (L ⊗ I)|Φ_d⟩*, where *Tr L^{†}L = d*, hence *‖L‖_{∞}^2 ≤ d*. Multiplication of the Choi difference on both sides by *L ⊗ I* bounds its trace norm by *d* times the Choi trace norm. Convexity handles mixed inputs. The last assertion follows from postprocessing contraction and the reverse triangle inequality.

Assumption. For a complete finite interface, the Stinespring-continuity consequence used here is, in full diamond normalization,

$$\frac{1}{4}\varepsilon_{\mathrm{raw}}^2 \le \delta_{\mathrm{raw}} \le 2\sqrt{\varepsilon_{\mathrm{raw}}},$$

where

$$\varepsilon_{\mathrm{raw}} = \inf_{\mathscr{D}} \|\mathscr{D}\mathscr{N} - \mathrm{id}_C\|_{\diamond}, \qquad \delta_{\mathrm{raw}} = \inf_{\tau} \|\mathscr{C} - \mathscr{T}_{\tau}\|_{\diamond}.$$

This is the explicitly imported approximation theorem [KSW].

Corollary. Under Assumption ,

$$\varepsilon_{\mathrm{rec}}(\mathscr{N}) \le \min\{a_d, \sqrt{2\delta_{\mathrm{full}}(\mathscr{C})}\}, \qquad \delta_{\mathrm{full}}(\mathscr{C}) \le \min\{1, \sqrt{2\varepsilon_{\mathrm{rec}}(\mathscr{N})}\}.$$

Proof. Substitute *ε_raw = 2ε_rec* and *δ_raw = 2δ_full*. The independent cap is Lemma .

A nearly constant complete complement gives an upper recovery bound. A nearly constant accessible output gives a lower recovery bound. The factors and directions cannot be exchanged.

The complete classical cut

Definition. The complete accessible output is classical-only if its algebra is the diagonal algebra of a finite history *Y*, including every declared outcome, timing, failure, control, and heat-meter label. No receiver memory, quantum bath, shared entanglement across the cut, or other input-bearing quantum factor is available to its decoder.

Lemma. Every finite adaptive process with exclusively classical accessible terminal output has the form

$$\mathscr{M}(X) = \sum_y \mathrm{Tr}\,(F_y X)\mathrm{proj}\, y, \qquad F_y \ge 0, \quad \sum_y F_y = I_C.$$

Proof. Compose the CP maps along each complete history and trace every inaccessible quantum output. Its probability is a positive linear functional on *ℬ(C)*, hence *Tr (F_yX)* for a positive effect. Completeness, with stopping and failure branches retained, gives *∑_y F_y = I_C*.

Theorem. For every finite classical-output channel *𝒩* and every deterministic decoder *𝒟: Y → C*,

$$F_e(\mathscr{D}\mathscr{M}) := \mathrm{Tr}\,[\Phi_d(\mathrm{id}_Q \otimes \mathscr{D}\mathscr{M})(\Phi_d)] \le \frac{1}{d},$$

$$d_\diamond(\mathscr{D}\mathscr{M}, \mathrm{id}_C) \ge b_d, \qquad \varepsilon_{\mathrm{rec}}(\mathscr{M}) \ge b_d.$$

The constants are sharp over this class. For the distinct Haar-average pure-state fidelity,

$$F_{\mathrm{av}}(\mathscr{D}\mathscr{M}) \le \frac{2}{d+1}, \qquad 1 - F_{\mathrm{av}}(\mathscr{D}\mathscr{M}) \ge \frac{d-1}{d+1}.$$

If an implemented channel $\mathscr{N}$, on the same complete output space, satisfies $d_{\diamond}(\mathscr{N}, \mathscr{M}) \le \delta$, then

$$\varepsilon_{\mathrm{rec}}(\widetilde{\mathscr{N}}) \ge [b_d - \delta]_+.$$

Proof. Let $\omega_y = \mathscr{D}(\mathrm{proj}\, y)$. Then

$$\mathscr{D}\mathscr{M}(X) = \sum_y \mathrm{Tr}\,(F_y X)\omega_y$$

is measure-and-prepare. Its Bell-input state is separable. For product states a, b,

$$\mathrm{Tr}\,(\Phi_d(a \otimes b)) = \frac{1}{d}\mathrm{Tr}\,(a^{\mathrm{T}} b) \le \frac{1}{d}.$$

Convexity proves . Testing the Bell projector gives referenced trace distance at least $1 - F_e$, proving .

Computational readout followed by basis-state preparation gives dephasing

$$\Delta(X) = \frac{1}{d}\sum_{k=0}^{d-1} Z^k X Z^{-k}, \qquad Z = \sum_{j=0}^{d-1} e^{2\pi i j/d}\mathrm{proj}\, j.$$

Separating the identity term gives $d_{\diamond}(\Delta, \mathrm{id}_C) \le 1 - 1/d$; the Bell lower bound gives equality and $F_e(\Delta) = 1/d$.

The pure-state second moment

$$\int \mathrm{proj}\,\psi^{\otimes 2}\, \mathrm{d}\psi = \frac{I + F}{d(d+1)}$$

implies

$$F_{\mathrm{av}}(\mathscr{T}) = \frac{1 + dF_e(\mathscr{T})}{d+1}.$$

This proves , again attained by dephasing. Finally use recovery stability in Lemma .

The entanglement-breaking mechanism and fidelity relation are standard [HSR,NielsenAverage,MassarPopescu]. The theorem is the common metric content of I-C Theorems A and D and Hawk05-A's classical obstruction [CheungIC,HawkA].

Remark. The theorem holds for every admitted channel and decoder, with the same Bell-input witness. It does not state that every input state suffers error b_d. Known basis states can be read and re-prepared perfectly. The whole-code requirement is correctness on all referenced inputs, and the diamond metric tests that requirement.

Classical heat labels and arbitrary finite precision only enlarge the POVM history. They do not evade the theorem. Coherent access to the reservoir is a different premise. Likewise, a finite outcome alphabet can have an injective exact probability law on a continuum of prescribed parameters; an exact law is not one realized finite record.

Corollary. For $d \ge 2$, a classical-only accessible endpoint cannot have a constant complete complement, even after every named residual has been consumed.

Proof. A constant complete complement would imply exact recovery by Theorem , contradicting Theorem .

Classical control with a surviving quantum carrier

Theorem. Let

$$\mathscr{H}(X) = \sum_y \mathrm{proj}\, y_Y \otimes F_y(X)_K,$$

where F_y are CP and $\sum_y F_y$ is TP. Then $\mathscr{H}$ exactly recovers the whole code if and only if:

• $\mathrm{Tr}\, F_y(X) = p_y \mathrm{Tr}\, X$ for input-independent probabilities p_y;

• whenever $p_y > 0$, the channel F_y/p_y has an exact decoder on the surviving quantum carrier K.

Proof. A dilation of the classical output can keep a copy of y in its environment. Exact recovery makes that complete environment constant by Theorem ; reading the copy proves (i). Restrict an exact decoder to each diagonal y-block. The resulting channels obey

$$\sum_{y\,:\,p_y>0} p_y\,\mathcal{D}_y(F_y/p_y) = \mathrm{id}_C.$$

The identity Choi operator has rank one. Positivity makes every normalized summand the identity channel, proving (ii). Conversely, read y and apply its branch decoder. A zero-probability CP branch has zero effect and is identically zero.

Thus a correction label can assist quantum recovery without being the quantum code. An input-informative retained classical outcome, however, is incompatible with exact whole-code recovery in this hybrid setting.

Cumulative records, endpoint choices, and geometric reports

Theorem. Fix an input state and the unconditioned law. For an unedited cumulative classical history $Y_{\le n} = (Y_1, \ldots, Y_n)$,

$$H(Y_{\le n+1}) = H(Y_{\le n}) + H(Y_{n+1} \mid Y_{\le n}) \ge H(Y_{\le n}).$$

Equality holds exactly when the appended value is almost surely a function of the old history. For a retained quantum factor K_n,

$$S(Y_{\le n}K_n) = H(Y_{\le n}) + \sum_y p_y S(\rho_{K_n \mid y}) \ge H(Y_{\le n}).$$

If no quantum factor is returned, every accessible persistent observable lies in the commutative record algebra.

Proof. Trace preservation preserves the old-history marginal. Apply the Shannon chain rule; finite classical conditional entropy is nonnegative and vanishes exactly for point-mass conditional laws. For , the eigenvalues of a block-diagonal state are p_y times those of its normalized blocks. The final assertion follows from the declared diagonal algebra.

This is the record result of I-C Theorems B and F [CheungIC]. It is not a Page curve of unmeasured radiation. For a separate classical message L, the quantity $H(L \mid Y_{\le n})$ can decrease while $H(Y_{\le n})$ does not.

Proposition. A residual-only trace-preserving operation, with all its new outputs ignored, leaves the pre-existing accessible channel unchanged. In particular, residual Enter followed by classical discard equals partial trace and changes neither that channel nor its recovery error.

Proof. The adjoint of a residual channel is unital, so every observable $A_R \otimes I_B$ is unchanged in expectation. Equation gives the stated destructive instance, including references.

Theorem. A finite prefix $\mathcal{P}: C \to Y \otimes K$, with classical Y and a residual K of unspecified terminal ownership, does not select a unique terminal access specification. The following all preserve the old Y-law:

• retain K as inaccessible;

• destructively read K, append its outcomes, and return no live K-name;

• grant quantum access to K. The third output is generally classical–quantum. After an actual complete classical cut, further classical-only processing cannot universally recover the unknown code. No choice of vacuum, remnant, final burst, or terminal geometry follows from the prefix.

Proof. Each continuation is trace preserving before ownership restriction, so its new outputs sum or trace to the same old marginal. The interfaces differ in access or appended data. The first and third may even have the same unrestricted factor map while assigning different ownership. After a complete classical cut, postprocessing and state preparation remain measure-and-prepare, so Theorem applies. The prefix contains no terminal Hamiltonian or geometry-update law.

This is the finite endpoint meaning of I-C Theorem E [CheungIC]. Consuming a name also does not preclude prior logical transfer: SWAP an unknown state into a surviving register and then destructively read the now-blank original name.

Geometric sensitivity is a different target

I-B-ECHO and I-C describe image-isolated scalar windows whose supplied echo functional distinguishes prescribed deficits, with short-window injectivity on compact non-shadow sectors under their stated kernel, gate, normalization, and preparation assumptions [CheungEcho,CheungIC]. Those are documentary comparisons here, not imported field-measurement theorems.

The same records, if they form the complete accessible output of an unknown-code task, remain subject to Theorem . Learning a deficit and returning an arbitrary code are different targets.

The neutral-core result in I-C concerns convergence of fixed smeared reports under a specified nonnegative smooth-core family [CheungIC]. Its statistical consequence has the following general finite form.

Lemma. Suppose report vectors $z_{\varepsilon} \to z_0$ and a fixed joint preparation rule $z \mapsto \rho(z)$ is trace-norm continuous. After one common finite instrument, the classical laws satisfy $TV(P_{\varepsilon}, P_0) \to 0$. For any fixed number n of independent repetitions,

$$\mathrm{TV}\left(P_{\varepsilon}^{\otimes n}, P_0^{\otimes n}\right) \le n\,\mathrm{TV}\left(P_{\varepsilon}, P_0\right) \to 0.$$

Thus no fixed-budget equal-prior test has success bounded above $1/2$ by a uniform positive margin along that limit.

Proof. Preparation continuity and instrument contraction prove the first claim. Telescoping tensor products gives the second. The optimal equal-prior classical testing success is $(1 + TV)/2$.

The lemma does not derive the continuum convergence hypotheses. Nor does convergence imply equality or non-injectivity of exact real functionals. Discontinuous rounding requires a separate threshold margin. These are precisely the finite-precision boundaries relevant to the shredder interpretation.

A corrected conserving single-excitation calibration

The coherent schedule

Give a memory M and each outgoing cell R_j the space

$$\mathbb{C}|\varnothing\rangle \oplus \mathbb{C}^d.$$

Initially M contains the code and the cells are vacant. For $0 \le \theta_j \le \pi/2$, let a unitary rotate each pair

$$|i, \varnothing\rangle \mapsto \cos\theta_j |i, \varnothing\rangle + \sin\theta_j |\varnothing, i\rangle,$$

with the complementary orthogonal rotation on $|\varnothing, i\rangle$, and identity elsewhere. It commutes with

$$H_0 = \omega\Big(P_M + \sum_j P_{R_j}\Big), \qquad \omega > 0,$$

because the two rotated vectors have equal energy.

Set

$$q_n = \prod_{j=1}^{n} \cos^2\theta_j, \qquad q_0 = 1, \qquad p_j = q_{j-1} - q_j.$$

Up to fixed vacant spectators, the complete isometry is

$$V_n|\psi\rangle = \sqrt{1 - q_n}\, J_n|\psi\rangle_R|\varnothing\rangle_M + \sqrt{q_n}\,|\varnothing\rangle_R|\psi\rangle_M,$$

where

$$J_n|i\rangle = \frac{1}{\sqrt{1 - q_{nj}}} \sum_{j=1}^{n} \sqrt{p_j}|j, i\rangle$$

when $q_n < 1$. At $q_n = 1$ the emitted term is zero.

Theorem. Writing $q = q_n$,

$$\mathcal{N}_q(X) = (1 - q) J_n X J_n^{\dagger} + q\,\mathrm{Tr}\,(X)\,\mathrm{proj}\,\varnothing_R,$$
$$\mathcal{C}_q(X) = qX + (1 - q)\,\mathrm{Tr}\,(X)\,\mathrm{proj}\,\varnothing_M,$$

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}_q) = qa_d, \qquad \delta_{\mathrm{full}}(\mathcal{C}_q) = qa_d, \qquad \varepsilon_{\mathrm{rec}}(\mathcal{C}_q) = (1-q)a_d,$$
$$S(\mathcal{N}_q(\rho)) = h_2(q) + (1-q)S(\rho), \qquad S(\mathcal{C}_q(\rho)) = h_2(q) + qS(\rho).$$

For a pure referenced input with code marginal ρ_C,

$$I(Q:R) = 2(1-q)S(\rho_C), \qquad I(Q:M) = 2qS(\rho_C).$$

Proof. Tracing the orthogonal memory or radiation flags gives . Decode the occupied radiation block by *J_n^{†}*, and prepare *I_C/d* on vacancy. The decoded channel is *(1 − q)id_C +q𝒯_{I_C/d}*, giving the upper bound *qa_d*. On a Bell input any decoder has Bell overlap at most *1 − q + q/d^2*, giving the matching lower bound. Interchanging weights gives residual recovery.

For leakage, compare with *qI_C/d ⊕ (1 − q)proj ∅*, giving the upper bound *qa_d*. For the lower bound, pinch any candidate replacement state into occupied and vacant blocks and average its occupied block over logical unitaries. Channel covariance and convexity permit a candidate *sI_C/d ⊕ (1 − s)proj ∅*. On the Bell input, twice the trace distance is

$$\left|q - s/d^2\right| + (d^2-1)s/d^2 + |s-q|.$$

If $s \le q$, this is $2q - 2s/d^2 \ge 2qa_d$. If $q \le s \le qd^2$, it is $2sa_d \ge 2qa_d$. If $s \ge qd^2$, it is $2(s-q) \ge 2qa_d$. The matching comparison has $s = q$.

The entropy formulas follow from orthogonal blocks. For a pure referenced input, *S(QR) = h_2(q) + qS(ρ_C)*. Combine this with *S(Q) = S(ρ_C)* and to obtain the mutual informations.

The reversed formula *δ_full(𝒞_q) = (1 − q)a_d* is not retained. It is the residual *recovery* error that has that weight, as distinguished in the supplied Hawk05 corrections [HawkA,HawkB].

The complete destructive history

Immediately read every returned cell in its occupied logical basis and vacancy basis, retaining every outcome. The complete history is either no emission, denoted *e*, or an emission time *j* and logical label *i*. For $u_i = \langle i|\rho|i\rangle$,

$$P(e) = q_n, \qquad P(j,i) = p_j u_i.$$

Theorem. The classical history channel $\mathcal{M}_n$ in satisfies

$$\varepsilon_{\mathrm{rec}}(\mathcal{M}_n) = q_n a_d + (1-q_n)b_d,$$
$$H_n = H(p_1, \ldots, p_n, q_n) + (1-q_n)H(u).$$

If $q_{n-1} > 0$ and $t_n = q_n/q_{n-1}$, then

$$H_n - H_{n-1} = q_{n-1}h_2(t_n) + (q_{n-1} - q_n)H(u) \ge 0.$$

If $q_{n-1} = 0$, the increment is zero.

Proof. The history is a basis measurement with total weight $1 - q_n$, plus an input-independent erasure outcome of weight q_n. Preparing the observed basis state or *I_C/d*, respectively, gives the upper bound in . On the Bell input, the total Bell overlap is at most $(1-q_n)/d + q_n/d^2$, proving the matching lower bound.

Equation is the entropy of . At the next step the old no-emission outcome of weight q_{n-1} splits into weights p_n and q_n; an emission also reveals a logical label with law u. The entropy chain rule gives .

A coarse snapshot has entropy

$$h_2(q_n) + (1-q_n)H(u),$$

not . Their difference is

$$(1-q_n)\,H\!\left(\frac{p_1}{1-q_n}, \ldots, \frac{p_n}{1-q_n}\right)$$

when $q_n < 1$, although their recovery optima coincide.

For $\theta_1 = \pi/4$, $\theta_2 = \pi/2$ and a basis input, coherent radiation entropies are *(0, 1, 0)*, whereas cumulative classical-history entropies are *(0, 1, 1)*. For pure unreferenced inputs the coherent entropy is *h_2(q_n)*, at most one bit. This is not an extensive Page law. A discrete schedule can skip $q = 1/2$, even jumping directly from one to

zero.

Finite Page conversion: one Haar device and two return types

Nested coherent release and the two exact entropy means

One fixed device and a conserving realization

Choose integers $s_1, \ldots, s_N \geq 2$, a final core dimension $b_{*} \geq 1$, and

$$H_{\mathrm{int}} = \left(\mathrm{bigotimes}_{j=1}^{N} \mathbb{C}^{s_j}\right) \otimes \mathbb{C}^{b_*}, \qquad D = b_* \prod_{j=1}^{N} s_j.$$

Fix an isometry $J: C \to H_{int}$, $d \leq D$. Choose one unitary $U \in U(D)$, independent of the logical input. It remains fixed at every cut and is known when a decoder is invoked.

For ensemble statements only, U is Haar distributed. This is an additional hypothesis. It is not derived from Enter, the cone, a vacuum state, a current, or energy conservation.

Release cells in order, leaving earlier quantum outputs untouched. At cut k,

$$r_k = \prod_{j=1}^{k} s_j, \qquad b_k = b_* \prod_{j=k+1}^{N} s_j, \qquad r_k b_k = D.$$

The complete isometry, suppressing fixed blanks, is

$$V_k^U = F_k U J : C \to R_k \otimes B_k,$$

where F_k is tensor regrouping.

For a conserving register implementation, give each memory and outgoing cell a vacant sector and an occupied logical sector. Choose matching memory/outgoing gaps $\omega_j > 0$, and declare

$$H_0 = \sum_j \omega_j (P_{M_j} + P_{R_j})$$

with any retained core energy included separately. The all-occupied internal sector is degenerate for this Hamiltonian, so U extended by identity commutes with H_0. Cell-by-cell SWAP also commutes with H_0. This is an exact register conservation law, not a gravitational mass law.

The Haar operation may be specified at one co-located preparation laboratory. It is not an instantaneous nonlocal operation on a spatially separated field. Packet identification, routing, and receiver implementation remain separate entries of the host contract.

Definitions of the two means

Let

$$\mathrm{H}_{\ell} = \sum_{j=1}^{\ell} \frac{1}{j}, \qquad m = \min(r, b), \quad n = \max(r, b).$$

Define

$$\mathrm{P}(r, b) = \frac{\mathrm{H}_{rb} - \mathrm{Harm}\, n - (m-1)/(2n)}{\ln 2},$$

$$\mathrm{L}(r, b) = \frac{\mathrm{H}_{rb} - \mathrm{Harm}\, b}{\ln 2}.$$

Theorem. For every fixed pure code input independent of U,

$$\mathbb{E}_U S(R_k) = \mathrm{P}(r_k, b_k), \qquad \mathbb{E}_U H(Y_{\leq k}) = \mathrm{L}(r_k, b_k),$$

where $Y_{\leq k}$ is the complete product-basis record obtained by immediately destructively reading each released cell.

The record mean increases strictly at every nontrivial release. The coherent mean increases at consecutive cuts remaining on the $r_k \leq b_k$ side, and decreases at consecutive cuts remaining on the $r_k \geq b_k$ side. If the crossing is traversed, a maximal mean occurs at one of the two cuts bracketing it.

If *b_{*} = 1*, the final coherent mean is zero, whereas

$$\mathbb{E}_U H(Y_{\le N}) = \frac{\operatorname{Harm} D - 1}{\ln 2} > 0.$$

No turnover is guaranteed if the schedule stops before the crossing.

Proof. Haar invariance makes $UJ|\psi\rangle$ a uniform pure vector in $\mathbb{C}\text{^}D$. Appendix evaluates its reduction entropy as ; this is the Page–Sen formula [PageAverage,Sen].

Write the vector's product-basis coordinates as independent complex Gaussians divided by their joint norm. A fixed radiation-prefix outcome collects *b_k* squared Gaussian coordinates. Its *r_k* probabilities therefore have the Dirichlet law with all parameters *b_k*, and

$$\mathbb{E}\sum_y p_y^t = r_k \frac{\Gamma(b_k + t)\Gamma(D)}{\Gamma(b_k)\Gamma(D + t)}.$$

Differentiating at *t = 1* gives . Later release operations act only on unmeasured cells, so immediate readout has exactly these joint prefix probabilities.

The record increment is

$$\frac{\mathrm{H}_{b_{k-1}} - \mathrm{H}_{b_k}}{\ln 2} > 0.$$

For an early-side coherent step, write dimensions as *m*, *n* and transfer a factor $s \ge 2$, giving *sm*, *n/s*, still with $sm \le n/s$. Its Page-mean increment in natural units is

$$\sum_{j = n/s + 1}^{n} \frac{1}{j} - \frac{(s^2 - 1)m - (s - 1)}{2n}.$$

Since $n \ge s\text{^}2m$, this is at least

$$1 - \frac{1}{s} - \frac{s^2 - 1}{2s^2} = \frac{(s - 1)^2}{2s^2} > 0.$$

Exchange the factors for the late side. The endpoint values follow by substitution.

The theorem locally establishes the common finite targets of Hawk05-B and Hawk05-C [HawkB,CheungHawkC]. The entropy in the second formula is the unconditioned Shannon entropy of the entire retained cell record, not uncertainty after observing one particular history.

Remark. The exact formulas use deterministic preinstalled timing labels and ideal zero-probability failure symbols. They therefore also describe the full ideal accessible classical history when those labels are appended. Additional random clock or detector records must be included in the joint law; they cannot be assumed to leave the exact mean unchanged.

Finite deficits and one-sample all-prefix control

Proposition. Put

$$T(r, b) = \log_2 \min(r, b), \qquad u(r, b) = \log_2 \left(1 + \frac{\min(r, b)}{\max(r, b)}\right).$$

Then

$$0 \le T(r, b) - \mathrm{P}(r, b) \le u(r, b),$$
$$0 \le \log_2 r - \mathrm{L}(r, b) \le \frac{1}{b \ln 2}.$$

For one sampled Haar history and any $z > 0$,

$$\mathbb{P}\left\{\max_k \left[T(r_k, b_k) - S(R_k)\right] > z\right\} \le \min\left\{1, \frac{\sum_k u(r_k, b_k)}{z}\right\}.$$

Proof. The Haar two-copy identity

$$\mathbb{E}\operatorname{proj} v^{\otimes 2} = \frac{I + F}{D(D + 1)}$$

gives

$$\mathbb{E}\operatorname{Tr}\rho_m^2 = \frac{m+n}{mn+1}.$$

The rank bound gives *S(ρ_m) ≤ log_2 m*. Jensen's inequality applied to *S(ρ_m) ≥ −log_2 Tr ρ_m^2* gives

$$\log_2 m - \mathbb{E}S(\rho_m) \le \log_2\left(\frac{m(m+n)}{mn+1}\right) \le \log_2(1+m/n).$$

For the record, harmonic-sum integral bounds give

$$\ln\frac{D+1}{b+1} \le \operatorname{Harm} D - \operatorname{Harm} b \le \ln\frac{D}{b}.$$

Their deficit from *ln r* is at most *ln (1 + 1/b) ≤ 1/b*. Finally, all sample deficits are nonnegative. Their maximum is bounded by their sum; Markov's inequality proves without independence between cuts.

Corollary. For *s_j = s ≥ 2*, *b_{*} = 1*, and *N* cells, define

$$C_s = \frac{s^2+1}{(s^2-1)\ln 2}.$$

Then

$$\mathbb{P}\left\{\max_{0\le k\le N}[\min(k,N-k)\log_2 s - S(R_k)] > z\right\} \le \min\{1, C_s/z\}.$$

Consequently, for fixed *s*, the normalized entropy history converges in probability to the capacity tent as *N → ∞*, for any sequence of fixed pure inputs independent of the sampled unitaries.

Proof. Use *u ≤ s^{−|N − 2k|}/ln 2*. For even *N*, the infinite extension of the geometric sum is *1 + 2/(s^2 − 1)*; for odd *N*, it is *2s/(s^2 − 1)*, no larger. Apply , then take *z = εNlog_2 s*.

This is not a uniform assertion over all inputs chosen after *U*. For a full code, the *U*-dependent input *U^{†}|0, …, 0⟩* has zero entropy at every cut.

Averaging and the marginal coherence gap

Proposition. For a fixed pure input,

$$\mathbb{E}_U\rho_{R_k}^U = \frac{I_{R_k}}{r_k}, \qquad S(\mathbb{E}_U\rho_{R_k}^U) = \log_2 r_k.$$

If the unitary is discarded and the accessible channel is averaged,

$$\mathbb{E}_U\mathcal{N}_k^U(X) = \operatorname{Tr}(X)\frac{I_{R_k}}{r_k},$$

which is replacement.

For any fixed radiation state and declared dephasing basis,

$$H(Y_{\le k}) - S(\rho_{R_k}) = S(\Delta_{R_k}\rho_{R_k}) - S(\rho_{R_k}) = D_2(\rho_{R_k}\|\Delta_{R_k}\rho_{R_k}) \ge 0.$$

Proof. Unitary twirling gives the first and second channel identities. For , the logarithm of the dephased state is diagonal, so

$$\operatorname{Tr}(\rho\log_2\Delta\rho) = \operatorname{Tr}(\Delta\rho\log_2\Delta\rho).$$

Positivity gives the required support inclusion, and relative entropy is nonnegative.

The Page quantity is *𝔼_US(ρ_R^U)*, not *S(𝔼_Uρ_R^U)*. Equation concerns one marginal and one basis. It is neither complete complementary leakage nor a thermodynamic cost. For example, an identity channel's Bell-input marginal is *I/d*, with zero marginal gap, although destructive basis readout still has whole-code recovery error *b_d*.

Whole-code recovery is not a Page mean

The full-code calibration

Theorem. If *d = D*, so *J* is unitary, then for every *U* and every cut with dimensions *r*, *b*,

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}_k^U) = 1 - b^{-2}.$$

Thus a fixed-pure-input Page mean does not imply small whole-code recovery error.

Proof. Absorb *UJ* into the logical identification. The channel is *Tr_B* on $R \otimes B$. The decoder appending *I_B/b* has error $d_{\{\diamond\}}(id_R \otimes \mathcal{T}_{I_B/b}, id_{RB}) = 1 - b^{\{-2\}}$.

For the reverse bound, use the input $\Phi_{r_Q_RR} \otimes \Phi_{b_Q_BB}$. After tracing *B*, Q_B is independent of every decoder output. The Bell overlap on Q_{BB_out} is therefore $b^{\{-2\}}$. Testing that Bell projector gives error at least $1 - b^{\{-2\}}$. Undo the logical identification.

For the full-code Bell input, the logical marginal is *I_D/D*; hence $S(R_k) = \log_2 r_k$, not the pure-unreferenced Page history. Preserving entanglement with *Q* does not make the radiation marginal pure.

A conservative Haar subcode certificate

At one cut let

$$\omega_{QB}^U = (\mathrm{id}_Q \otimes \mathcal{C}^U)(\Phi_d), \qquad \sigma_B^U = \mathcal{C}^U(I_C/d),$$

and put

$$\gamma(r,b,d) = \frac{(1-d^{-2})r(b^2-1)}{(rb)^2-1}, \qquad \Lambda(r,b,d) = \frac{d}{2}\sqrt{db\,\gamma(r,b,d)}.$$

Theorem. For $D = rb \geq 2$,

$$\mathbb{E}_U \left\| \omega_{QB}^U - I_Q/d \otimes \sigma_B^U \right\|_2^2 = \gamma(r,b,d).$$

Consequently

$$\mathbb{E}_U \delta_{\mathrm{full}}(\mathcal{C}^U) \leq \min\{1, \Lambda(r,b,d)\}.$$

For $0 < \zeta < 1$, with probability at least $1 - \zeta$,

$$\delta_{\mathrm{full}}(\mathcal{C}^U) \leq \min\{1, \Lambda/\zeta\}, \qquad \varepsilon_{\mathrm{rec}}(\mathcal{N}^U) \leq \min\{a_d, \sqrt{2\Lambda/\zeta}\}.$$

For fixed *d*, this certificate vanishes when $b/r \to 0$. The sufficient growing-code regime is $d^3 b/r \to 0$.

Proof. Appendix proves the Hilbert–Schmidt identity. The operator acts on a *db*-dimensional space, so

$$\tfrac{1}{2}\|\omega - I_Q/d \otimes \sigma\|_1 \leq \tfrac{1}{2}\sqrt{db}\,\|\omega - I_Q/d \otimes \sigma\|_2.$$

The right Choi bound in and Jensen's inequality give the mean leakage estimate. Markov's inequality gives the leakage event, and Corollary gives recovery on that same event. The asymptotic statements follow from . They are conservative, not optimal random-code thresholds.

For several cuts, a union bound with predeclared failure budgets gives a simultaneous certificate. Alternatively, passive nested access allows a certified earlier decoder to be used later by discarding new outputs, as proved in Section .

An extensive entropy history without code release

Theorem. Assume $b_{\{*\}} = 1$. There are two conserving sequential complete interfaces with the same accessible and residual spaces such that:

• for one fixed pure code input, their accessible states agree at every prefix for the same sampled *U*;

• for every fixed pure input independent of *U*, their ensembles have the same exact Page means;

• their endpoint recovery errors are *0* and a_d, and their endpoint complete leakages are *0* and a_d.

Proof. Append a degenerate inaccessible register $L \simeq C$. In the returning model, run the cell-release model on $J|\psi\rangle$ and leave *L* fixed in $|0\rangle$. In the retaining model, store $|\psi\rangle$ in *L* and run the same release schedule on $J|0\rangle$. Both are isometries into the same spaces and conserve the declared register Hamiltonian.

For input $|0\rangle$, the radiation states agree at every cut for the same *U*. Haar invariance gives the same Page mean for any fixed pure input in the returning model and the dummy input in the retaining model.

At full release the returning radiation contains $UJ|\psi\rangle$ with a fixed complete residual, so recovery and leakage are zero. The retaining radiation is fixed at $UJ|0\rangle$, while the residual contains an isometric copy of $|\psi\rangle$. Its radiation recovery error is a_d by Lemma . Compressing the residual to the code and comparing with the embedded maximally mixed state gives leakage exactly a_d.

This is an explicit model witness for Theorem , and a locally proved form of the Hawk05-B retaining-memory comparison [HawkB]. It does not compare two models under an identical assumption of complete outgoing access and a unique empty physical endpoint.

Proposition. Let σ_K have purification $|\chi\rangle_{KP}$, and let $L \simeq C$. The isometries

$$V_g|\psi\rangle = |\psi\rangle_{C_R}|\chi\rangle_{KP}|0\rangle_L, \qquad V_b|\psi\rangle = |0\rangle_{C_R}|\chi\rangle_{KP}|\psi\rangle_L$$

have identical accessible spectra for every pure input but recovery errors 0 and a_d. For $H_R = I_{C_R} \otimes H_K$, their energy-measurement laws agree. The state σ_K may be Gibbs.

Proof. The accessible channels are $X \mapsto X \otimes \sigma_K$ and $X \mapsto \mathrm{Tr}(X)\mathrm{proj}\,0 \otimes \sigma_K$. For pure X, both spectra are those of σ_K, with zeros added. Trace out K to decode the first channel; the second is replacement. The selected Hamiltonian acts trivially on C_R.

Related spectrum/recovery separations are documented in En07-C and Oai265-A [CheungEn07C,CheungOai265]. A static area-law record is not used as a temporal history.

Unknown code, named observable, and geometry

For a named effect $0 \le F \le I_C$, exact observable reconstruction can mean existence of an accessible effect E_R such that

$$\mathcal{N}^*(E_R) = F.$$

This is weaker than $\mathcal{D}\mathcal{N} = \mathrm{id}_C$.

For example, $V|i\rangle = |i\rangle_R|i\rangle_B$ permits exact basis-label observation, but dephases the unknown code. A successful named-effect record does not contradict Theorem .

The distinction is retained in Oai290-A and in the finite bounded-effect benchmark XVII-A [CheungOai290,CheungXVII]. The latter assumes its geometric representation and reconstructs one named observable; it is not metric reconstruction or holographic emergence.

Proposition. Suppose two commuting operator representations π_1, π_2 of $\mathcal{B}(C)$ both preserve the encoded subspace and obey

$$\pi_1(X)V = VX, \qquad \pi_2(X)V = VX \quad \text{for every} X \in \mathcal{B}(C).$$

Then $\mathcal{B}(C)$ would be commutative. Hence this is impossible for $d \ge 2$.

Proof. Commutation gives

$$VXY = \pi_1(X)\pi_2(Y)V = \pi_2(Y)\pi_1(X)V = VYX.$$

Multiply by $V^{\dagger}$.

Independent commuting matrix algebras can of course exist. The proposition excludes their both implementing the same unknown logical algebra on one code. It is not a gravitational complementarity theorem.

Conserved sectors: the correct direct-sum calculation

A prescribed line-bundle holonomy is not automatically an additive quantum charge. Nor does a nonzero ground-state current prove typicality. If a charge interpretation is used, its operator, superselection convention, finite truncation, and compatibility with the encoding must be supplied in addition to the I-B-VP field data.

Assumption. At a given cut the admitted finite sector is

$$H_q = \mathrm{bigoplus}_{\lambda \in \mathcal{I}} H_{R,\lambda} \otimes H_{B,q-\lambda},$$

where distinct residual charge spaces are orthogonal,

$$r_\lambda = \dim H_{R,\lambda} \ge 1, \qquad b_\lambda = \dim H_{B,q-\lambda} \ge 1.$$

Let

$$D_\lambda = r_\lambda b_\lambda, \qquad D_q = \sum_\lambda D_\lambda, \qquad w_\lambda = D_\lambda / D_q.$$

The encoding maps into *H_q*, and one unitary is Haar distributed on *H_q*, extended compatibly outside it.

Theorem. For a fixed pure input independent of the sector Haar unitary,

$$\mathbb{E}S(R) = \frac{1}{\ln 2}\Big[\mathrm{H}_{D_q} - \sum_\lambda w_\lambda \mathrm{H}_{n_\lambda} - \sum_\lambda w_\lambda \frac{m_\lambda - 1}{2n_\lambda}\Big],$$

$$\mathbb{E}H(Y) = \frac{1}{\ln 2}\Big[\mathrm{H}_{D_q} - \sum_\lambda w_\lambda \mathrm{H}_{b_\lambda}\Big],$$

where *m_{λ} = min (r_{λ}, b_{λ})* and *n_{λ} = max (r_{λ}, b_{λ})*. The readout resolves the radiation charge sector and a basis within it.

Proof. Independent Gaussian coordinates in the blocks show that squared block norms *p_{λ}* have Dirichlet parameters *D_{λ}*. Conditional normalized block directions are independent Haar vectors and independent of the weights. Tracing the orthogonal residual charge spaces gives

$$\rho_R = \mathrm{bigoplus}_\lambda\, p_\lambda \rho_{R,\lambda}, \qquad S(\rho_R) = H(p) + \sum_\lambda p_\lambda S(\rho_{R,\lambda}).$$

Dirichlet differentiation gives

$$\mathbb{E}H(p) = \frac{\mathrm{H}_{D_q} - \sum_\lambda w_\lambda \mathrm{H}_{D_\lambda}}{\ln 2}.$$

Add ∑_{λ} *w_{λ}P(r_{λ}, b_{λ})* to obtain .

Each radiation basis outcome in block *λ* collects *b_{λ}* Gaussian coordinates; its Dirichlet parameter is *b_{λ}*, repeated *r_{λ}* times. The same differentiation gives .

If the sector weights are instead fixed at supplied *p_{λ}*, the corresponding means are

$$H(p) + \sum_\lambda p_\lambda \mathrm{P}(r_\lambda, b_\lambda), \qquad H(p) + \sum_\lambda p_\lambda \mathrm{L}(r_\lambda, b_\lambda).$$

That is a different ensemble.

The unrestricted two-dimension formula must not be applied merely by replacing with aggregate dimensions. For instance, the one-excitation sector of *N* qubits has two blocks at a cut: *(r, b) = (1, N − k)* and *(k, 1)*. Each conditional block has zero pure-state reduction entropy. Its radiation entropy is only the binary charge-weight entropy, at most one bit, not an extensive *N*-qubit tent. The direct-sum correction is the sector issue identified in Hawk05-C [CheungHawkC].

Conditional extrapolation: capacity clocks, receivers, and ledgers

Clock replacement, capacity compatibility, and cutoff

The fixed-dimensional sum constraint

Theorem. For one fixed nested tensor-release model,

$$\log_2 r_k + \log_2 b_k = \log_2 D.$$

Suppose real target capacities *A(t), B(t)* are compared with its cuts by

$$|\log_2 r_{k(t)} - A(t)| \le e_R(t), \qquad |\log_2 b_{k(t)} - B(t)| \le e_B(t).$$

Then

$$|A(t) + B(t) - \log_2 D| \le e_R(t) + e_B(t).$$

In particular, for any two times,

$$|[A(t) + B(t)] - [A(s) + B(s)]| \le e_R(t) + e_B(t) + e_R(s) + e_B(s).$$

A time reparametrization cannot remove a violation of this constraint.

Proof. Take logarithms of *r_kb_k = D*, then apply the triangle inequality. Reparametrization changes the

indexing, not that algebraic identity.

This constraint is specific to the tensor-release model. A sector-resolved model instead requires the direct-sum data of Section ; it is not silently assigned a single tensor capacity pair.

A supplied inverse-square envelope

Independently of the conical metric, supply positive constants λ, γ, ξ, an initial mass label M_0, and a positive cutoff $M_{cut} < M_0$. In units chosen for this envelope, assume

$$\frac{\mathrm{d}M}{\mathrm{d}T} = -\frac{\lambda}{M^2}, \qquad B(M) = \gamma M^2, \qquad A(M) = \xi\gamma(M_0^2 - M^2).$$

No equality between this mass label and a variable of is imposed.

Proposition. Within the retained positive-cutoff interval,

$$M(T) = [M_0^3 - 3\lambda(T - T_0)]^{1/3},$$

$$T_{\mathrm{cut}} - T_0 = \frac{M_0^3 - M_{\mathrm{cut}}^3}{3\lambda}.$$

The envelope $F(T) = \min\{A(M(T)), B(M(T))\}$ has its formal crossover at

$$M_\times = M_0\sqrt{\frac{\xi}{1+\xi}},$$

$$T_\times - T_0 = \frac{M_0^3}{3\lambda}\left[1 - \left(\frac{\xi}{1+\xi}\right)^{3/2}\right].$$

The crossover belongs to the retained interval exactly when $M_{cut} \le M_{\times}$. A strictly decreasing retained branch requires a strict inequality.

The two capacities in can be exact capacities of a fixed tensor-release proxy on a nontrivial mass interval only if $\xi = 1$. More generally,

$$|\xi - 1|\,[B(M_0) - B(M)] \le w(M_0) + w(M), \qquad w = e_R + e_B.$$

Proof. Integrate $M^2\,dM = -\lambda\,dT$. Solving $A(M) = B(M)$ gives . The first target increases and the second decreases as M decreases, proving the cutoff statement. Finally,

$$A(M) + B(M) = \xi B(M_0) + (1 - \xi)B(M).$$

Apply Theorem at M_0 and M.

The inverse-square clock can reparametrize a compatible proxy. The ratio $\xi \ne 1$ changes the capacity assignment, not merely its clock. This is a mathematical compatibility test, not a restriction derived from Einstein dynamics.

One rounded two-curve realization

Take $\xi = 1$, $B_0 = \gamma M_0^2 \ge 2$, and define

$$n = \lfloor B_0 \rfloor, \qquad k(T) = \min\{n, \lfloor A(M(T)) \rfloor\}.$$

Use n qubit cells, one sampled Haar unitary, and release a cell when $k(T)$ increments.

Theorem. The construction is one nested history with

$$r(T) = 2^{k(T)}, \qquad b(T) = 2^{n - k(T)}, \qquad D = 2^n.$$

It obeys

$$|\log_2 r(T) - A(M(T))| \le 1, \qquad |\log_2 b(T) - B(M(T))| \le 1.$$

Its exact coherent and record means are

$$\overline{Q}(T) = \mathrm{P}(2^{k(T)}, 2^{n - k(T)}), \qquad \overline{L}(T) = \mathrm{L}(2^{k(T)}, 2^{n - k(T)}),$$

and

$$|\overline{Q}(T) - F(T)| \le 1 + \log_2\left(1 + 2^{-|n - 2k(T)|}\right),$$

$$\left|\bar{L}(T) - A(M(T))\right| \le 1 + \frac{2^{-(n-k(T))}}{\ln 2}.$$

If *B(M_cut) ≥ 1*, at least one residual qubit remains at the cutoff.

Proof. Before the cap in *k(T)*, radiation rounding is less than one and

$$n - k(T) - B(M(T)) = n - B_0 + A(M(T)) - \lfloor A(M(T)) \rfloor$$

lies between *−1* and *1*. After the cap, *A ≤ B_0 < n + 1* and *B = B_0 − A < 1*, giving the same bounds. The minimum is one-Lipschitz in the maximum norm, so the target tent differs from *log_2 min (r, b)* by at most one. Apply Proposition . If *B(M_cut) ≥ 1*, then *A(M_cut) ≤ B_0 − 1*, hence *k(T_cut) ≤ n − 1*.

The ideal release times are installed classical data. An I-B clock can mark each actual event with a deterministic, event-calibrated reading, or carry a separately declared history label. Successive clocks use fresh factors and the complete composition bound . They do not quantize the geometric time axis or eliminate external time.

If a remote receiver is intended, its transit times and causal margins must also fit the schedule. Equations – do not supply that receiver.

Proposition. If on the same mass interval

$$(1-\rho)\lambda/M^2 \le P_{\mathrm{act}}(M) \le (1+\rho)\lambda/M^2, \qquad 0 \le \rho < 1,$$

then the actual time to mass *M* lies between

$$\frac{M_0^3 - M^3}{3\lambda(1+\rho)} \quad \text{and} \quad \frac{M_0^3 - M^3}{3\lambda(1-\rho)}.$$

Separately, suppose the envelope *F* has an interior crossover and *sup_T |f(T) − F(T)| ≤ E*. Every attained global maximum $\hat{T}$ of *f* satisfies

$$\left|\hat{T} - T_{\times}\right| \le \frac{2E}{v}, \qquad v = \frac{2\gamma\lambda \min\{\xi, 1\}}{M_0}.$$

Proof. Integrate the reciprocal power bounds. The magnitudes of the early and late slopes of *F* are *2ξγλ/M* and *2γλ/M*, respectively, both at least *v*. Hence *F(T_{×}) − F(T) ≥ v|T − T_{×}|*. At a maximizing $\hat{T}$, *f(T̂) ≥ f(T_{×})*, so the left side is at most *2E*.

This controls a specified envelope's maximum, not the time of maximal complete leakage or the onset of quantum recovery.

Occupations, capacities, and conditional entropy transfer

What an occupation can and cannot determine

A declared stationary number density may have the schematic form

$$n_a(\omega; M) = \frac{\Gamma_a(\omega; M)}{e^{8\pi M\omega} - 1}, \qquad 0 \le \Gamma_a \le 1,$$

in an independently specified Schwarzschild comparison and unit convention [Hawking,CheungEm03A]. It is not a formula derived from .

If, as an additional assumption, *Γ_a(ω; M) = γ_a(Mω)*, with convergent mode sums, then

$$\sum_a g_a \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\, \omega n_a(\omega; M) = \frac{1}{M^2} \sum_a g_a \int_0^\infty \frac{\mathrm{d}x}{2\pi}\, \frac{x\gamma_a(x)}{e^{8\pi x} - 1}.$$

This change of variables can explain an inverse-square envelope. It does not determine cell dimensions or a coherent receiver. Applying a stationary formula along *M(T)* additionally requires a quasistatic comparison.

On the cone, I-B-VP occupations and stresses remain prescribed-background identities. They are not dynamical greybodies of an evaporating geometry [CheungVP,CheungVP4]. Finite propagation restricts a supplied propagation channel; it does not generate a capacity history.

Proposition. For *0 < u < 1*, the same bosonic number mean *u* is compatible with a two-level Bernoulli state and an infinite-support geometric state.

Proof. The states are

$$(1-u)\mathrm{proj}\,0 + u\,\mathrm{proj}\,1, \qquad \sum_{n=0}^{\infty}(1-z)z^{n}\mathrm{proj}\,n, \quad z=\frac{u}{1+u}.$$

Both have number expectation u, but their supports differ.

For the geometric state, truncation above L has tail probability $z\hat{}\{L+1\}$. A finite readout can retain that tail as a failure symbol. A state-relative tail probability is not automatically a diamond bound on an arbitrary unknown code; such a bound needs a uniform code promise.

Finite continuity with explicit dimensions

For $L \geq 2$, define the monotone continuity modulus

$$c_L(x) = \begin{cases} h_2(x) + x\log_2(L-1), & 0 \leq x \leq 1-1/L, \\ \log_2 L, & 1-1/L \leq x \leq 1, \end{cases} \qquad c_1(x) = 0.$$

The finite entropy-continuity input is

$$d_{\mathrm{tr}}(\rho,\sigma) \leq x \quad \Rightarrow \quad |S(\rho)-S(\sigma)| \leq c_L(x)$$

on a common L-dimensional space [Audenaert]. The same modulus applies to classical laws on L labels.

Definition. At a fixed cut, suppose:

• the proxy unitary has distribution v, with $TV\,(v, v_Haar) \leq \tau$;

• actual and proxy accessible quantum outputs are identified in a common space of dimension L, preserving their access side;

• their channels differ by at most η in half-diamond distance, uniformly over the compared unitary labels;

• the actual complete detector differs from the declared destructive proxy detector by at most v_det;

• the joint classical comparison alphabet, including failures, has size K, and target capacities obey the rounding bounds of Theorem . The comparison is for one fixed input independent of the ensemble.

Theorem. Under Definition , let $m = min\,(r, b)$, $n = max\,(r, b)$ and $u = log_2\,(1 + m/n)$. Then

$$|\mathbb{E}S(R_{\mathrm{act}}) - \min(A,B)| \leq c_L(\eta) + \tau\log_2 m + u + \max(e_R, e_B),$$

$$|\mathbb{E}H(Y_{\mathrm{act}}) - A| \leq c_K(\min\{1, \eta + v_{\mathrm{det}}\}) + \tau\log_2 r + \frac{1}{b\ln 2} + e_R.$$

These entropy allowances do not certify whole-code recovery. Every actual record-only channel separately obeys Theorem .

Proof. Channel comparison controls every output trace distance by η. Composition and contraction bound the detector-law distance by $min\,\{1, \eta + v_det\}$. Apply and take expectations.

For any function in $[0, L_0]$, changing its measure by total variation at most τ changes its expectation by at most τL_0. Proxy coherent entropy lies in $[0, log_2\,m]$, and proxy record entropy in $[0, log_2\,r]$. Replace the proxy ensemble by Haar, then use Proposition . Finally,

$$|\min(\log_2 r, \log_2 b) - \min(A,B)| \leq \max(e_R, e_B).$$

The recovery statement is independent of these scalar estimates.

For a single sampled history, rather than its expectation, a separate sample-deviation event such as must be included. A mean comparison does not become a same-device pathwise theorem.

The supplied-source separation in Hawk05-C is retained: capacity, ensemble, channel, and detector allowances answer different questions [CheungHawkC]. A cutoff and a plotted pair of curves do not manufacture any of them.

Receiver and endpoint certificates

The four internal obligations

Definition. A same-model endpoint claim supplies:

• a justified finite unknown code, its prepared isometry, all ancillary purifications, and the external-reference

target;

• complete logical-input-independent evolution on the declared domain, including every unreturned output;

• access-preserving identification of the accessible and inaccessible sides separately, including a coherent receiver if quantum radiation is to be collected;

• a complete-complement replacement allowance on the entire code, or a decoded-channel allowance for a specified deterministic decoder.

This is the finite checklist documented in En07-B and En07-C [CheungEn07B,CheungEn07C]. The full-release proxy with $b_{*} = 1$, known U, and stipulated coherent register access satisfies it mathematically. No corresponding gravitational justification is supplied here.

Theorem. After the access-preserving identifications of Definition , suppose actual and proxy complete channels differ by at most κ in half-diamond distance. For a feasible proxy complementary state τ_B, let

$$e = \frac{1}{2}\| J_{\mathrm{n}}(\mathscr{C}_{\mathrm{pr}}) - I_Q/d \otimes \tau_B \|_1 .$$

Then

$$\delta_{\mathrm{full}}(\mathscr{C}_{\mathrm{act}}) \le \min\{1, de + \kappa\},$$
$$\varepsilon_{\mathrm{rec}}(\mathscr{N}_{\mathrm{act}}) \le \min\{a_d, \sqrt{2de} + \kappa\}.$$

The second bound is achieved as a bound for one fixed proxy decoder, unless the constant-decoder cap is smaller. If a specified proxy decoder $\mathscr{D}_{*}$ instead satisfies

$$r_* = \frac{1}{2}\| J_{\mathrm{n}}(\mathscr{D}_* \mathscr{N}_{\mathrm{pr}}) - \Phi_d \|_1 ,$$

then the same decoder on the actual accessible output obeys

$$d_{\diamond}(\mathscr{D}_* \mathscr{N}_{\mathrm{act}}, \mathrm{id}_C) \le \min\{1, dr_* + \kappa\}.$$

Terminal residual Enter followed by discard changes none of these pre-existing accessible channels.

Proof. Partial trace contracts the complete comparison, so accessible and complementary channels are each within κ. The right Choi bound in gives $d_{\diamond}(\mathscr{C}_{pr}, \mathscr{T}_{\tau_B}) \le de$. The triangle inequality proves . Corollary supplies a proxy decoder of error at most $\sqrt{2de}$; applying that fixed decoder to the actual channel adds at most κ. The constant decoder gives the alternative cap a_d. The same Choi bound and contraction prove . Proposition proves the final assertion.

The theorem is a deterministic implication from a complete-channel certificate. A finite-sample estimate requires a separately declared statistical model, calibration uncertainty, and coverage statement. The finite-certificate records Oai265-B and Oai227-A do not supply those assumptions for an unspecified receiver [CheungOai265B,CheungOai227]. In particular, the supplied Oai227-A comparison concerns a two-spin stationary–quench target; it is not an evaporation endpoint theorem.

A support-completed Petz candidate

Lemma. Let $\sigma_C > 0$ be a faithful state and $\tau_R = \mathscr{N}(\sigma_C)$, with support projection P. Inverse powers of τ_R are taken on P. For any fixed code state ω_C, define

$$\mathscr{R}_{\sigma}(Y) = \sigma_C^{1/2} \mathscr{N}^{*}(\tau_R^{-1/2} P Y P \tau_R^{-1/2}) \sigma_C^{1/2} + \mathrm{Tr}\,((I - P)Y)\omega_C .$$

Then $\mathscr{R}_{\sigma}$ is CPTP on all of $\mathscr{B}(R)$. It is a candidate decoder, not a recovery certificate. Its task error can be bounded by the decoded Choi discrepancy in .

Proof. Both summands are CP. Using adjoint duality, the trace of the first summand is

$$\mathrm{Tr}\left[\tau_R \tau_R^{-1/2} P Y P \tau_R^{-1/2}\right] = \mathrm{Tr}\,(PY).$$

The second trace is $\mathrm{Tr}\,((I - P)Y)$. Their sum is $\mathrm{Tr}\,Y$. The final assertion is Theorem , applied to this specified map.

The construction is related to the standard Petz recovery map [Petz]; naming it does not establish its error or its physical implementation. It is not an island decoder by definition. The named-observable result of XVII-A

remains a useful negative control against that inference [CheungXVII].

Nested access and the direction of processing

Proposition. Suppose

$$\mathcal{N}_k = \mathcal{L}_k \mathcal{N}_{k+1}$$

for a CPTP restriction $\mathcal{L}_k$. Then

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}_{k+1}) \le \varepsilon_{\mathrm{rec}}(\mathcal{N}_k).$$

For passive release acting only on the complete residual, $\mathcal{C}_{k+1} = \Lambda_k \mathcal{C}_k$, and

$$\delta_{\mathrm{full}}(\mathcal{C}_{k+1}) \le \delta_{\mathrm{full}}(\mathcal{C}_k).$$

By contrast, if $\mathcal{N}_{k+1} = \mathcal{L}_k \mathcal{N}_k$ is merely a degradation of the earlier output, the recovery inequality reverses.

Proof. A later decoder can first apply $\mathcal{L}_k$ and use any earlier decoder. For leakage, postprocess a replacement comparison:

$$d_\diamond(\Lambda_k \mathcal{C}_k, \mathcal{T}_{\Lambda_k(\tau)}) \mathrm{led}_\diamond (\mathcal{C}_k, \mathcal{T}_\tau).$$

Take infima. The degradation statement is the same decoder-inclusion argument in the other direction.

If earlier quantum outputs are measured, lost, or edited, the passive marginal relation may fail. Restricting access alone also does not imply an entropy tent; a product state can have zero entropy at every prefix.

A cutoff is not an autonomous endpoint

Proposition. Let H be a fixed finite-dimensional Hamiltonian and $V_T = e^{-iTH} V_0$ a fixed prepared complete interface. There are arbitrarily late times at which Ad_{V_T} is arbitrarily close to Ad_{V_0} in half-diamond distance. If the initial accessible channel is replacement, then

$$\limsup_{T \to \infty} \varepsilon_{\mathrm{rec}}(\mathcal{N}_T) = a_d.$$

Proof. There are finitely many eigenvalues. Simultaneous recurrence of their phases gives arbitrarily late T with $\|e^{-iTH} - I\|_\infty$ arbitrarily small. For isometries,

$$d_\diamond(\mathrm{Ad}_V, \mathrm{Ad}_W) \le \|V - W\|_\infty;$$

apply the pure-state trace-distance bound to referenced inputs and use convexity. Partial trace gives accessible-channel recurrence. Recovery stability and the universal cap a_d prove the limsup.

This does not apply to the externally stopped release schedule as an autonomous late-time claim, nor to a justified infinite outgoing scattering limit. Such limits need their own construction. Recurrence is not evaporation, and selecting a positive cutoff does not choose a remnant, a final burst, or an empty complete complement.

Thermal accounting and four independent ledgers

A channel is not its heat bill

Theorem. Let M contain all non-reservoir degrees of freedom in a finite implementation step, and let L be a reservoir with Hamiltonian H_L, initially Gibbs:

$$\tau_L = Z^{-1} e^{-\beta_{\mathrm{th}} H_L}, \qquad \beta_{\mathrm{th}} = (k_{\mathrm{B}} T)^{-1}, \quad T > 0.$$

Assume the complete initial state is $\rho_M \otimes \tau_L$ and the complete step is unitary. For final state ω_{ML}, define $Q_L = \mathrm{Tr}\,[H_L(\omega_L - \tau_L)]$. With natural-log entropy and relative entropy,

$$\beta_{\mathrm{th}} Q_L = S_{\mathrm{n}}(\rho_M) - S_{\mathrm{n}}(\omega_M) + I_{\mathrm{n}}(M:L)_\omega + D_{\mathrm{n}}(\omega_L \| \tau_L).$$

In particular,

$$Q_L \ge k_{\mathrm{B}} T [S_{\mathrm{n}}(\rho_M) - S_{\mathrm{n}}(\omega_M)].$$

Destructive readout alone implies no universal strictly positive heat cost per consumed name.

Proof. The Gibbs expression gives

$$D_{\mathrm{n}}(\omega_L \| \tau_L) = \beta_{\mathrm{th}} Q_L - S_{\mathrm{n}}(\omega_L) + S_{\mathrm{n}}(\tau_L).$$

Unitarity and initial independence give

$$I_{\mathrm{n}}(M : L)_{\omega} = S_{\mathrm{n}}(\omega_M) + S_{\mathrm{n}}(\omega_L) - S_{\mathrm{n}}(\rho_M) - S_{\mathrm{n}}(\tau_L).$$

Eliminate the reservoir entropy change. Nonnegativity of mutual information and relative entropy gives the inequality.

For the final assertion, implement the finite readout dilation by a unitary with pure blank apparatus registers, take all apparatus Hamiltonians zero, and leave an independent Gibbs reservoir uncoupled. The reservoir heat is exactly zero. This implementation consumes blanks and is not asserted to be cyclic. The abstract instrument does not specify their restoration.

The equality is the standard finite-reservoir Landauer balance [Landauer,ReebWolf], locally written to identify its assumptions. It is the thermal content of I-C Theorem C, not a consequence of *Tr_Y Enter = Tr_K* alone [CheungIC].

Corollary. Under Theorem , suppose initially *M = AZ* is classical, the record *Z* retains the same law, and *dim A = m ≥ 2*. If every supported final conditional *A*-state has reset-state probability at least *1 − δ*, where *0 ≤ δ ≤ 1 − 1/m*, then

$$Q_L \geq k_{\mathrm{B}} T[\,(\ln 2) H(A \mid Z) - h_{\mathrm{n}}(\delta) - \delta \ln (m-1)\,],$$

where *h_n(x) = −xln x − (1 − x)ln (1 − x)*. The bracket must be positive to force positive heat.

Proof. Dephase each final conditional state in a basis containing the reset state. Entropy cannot decrease under this pinching. At most *δ* probability lies in an *(m − 1)*-dimensional orthogonal subspace, so its entropy is at most *h_n(δ) + δln (m − 1)*. Average over *Z*, cancel its unchanged entropy, and apply Theorem .

The same reset channel can also have different energy transfers. For matched qubits, SWAP with a blank gives the same reduced reset for every gap *ε > 0*, but transfers energy *εp* when the initial excitation probability is *p*. Independent driven spectator operations can further change reservoir heat without changing the reported channel. Their resource restoration remains part of the implementation ledger.

Heating and failure are additional models

If a device has internal energy *U_dev(Θ)* with positive heat capacity *C_dev = U_dev^{'} > 0*, and a separately specified power balance

$$\frac{\mathrm{d}}{\mathrm{d}T} U_{\mathrm{dev}}(\Theta(T)) = P_{\mathrm{dep}}(T) - P_{\mathrm{out}}(T),$$

then

$$\dot{\Theta} = \frac{P_{\mathrm{dep}} - P_{\mathrm{out}}}{C_{\mathrm{dev}}(\Theta)}.$$

Cooling can balance deposited power. Reaching a specified thermal threshold requires the corresponding integrated net energy. Neither name consumption nor a recovery obstruction implies an explosion; material failure requires additional mechanical, chemical, and protection data [CheungIC].

Two ledgers of distinctions

The information-target ledger adopted from the documentary distinctions in Oai290-A is: toprule Target Required mathematical object Not supplied by

midrule Unknown-code recovery A deterministic reference-preserving decoder and a complete-channel error A Page mean, a thermal occupation, or a named-effect record

Named-observable reconstruction A specified effect or algebra and its accessible realization A successful whole-code label in another model

Geometric consistency A declared host, locations, causal embeddings, field domains, and any separately proved physical dictionary A code diagram, an entropy slope, or a decoder's name

bottomrule

The implementation ledger has four entries: toprule Ledger Data that must be stated

midrule Energy Complete Hamiltonians, controls, transfers, endpoint interaction energies, and work boundary

Reset and resources Blanks, fresh clocks, live-register peaks, external storage, classical records, failures, and cyclic restoration

Information leakage The complete complementary channel, unknown code, references, decoder domain, and half-diamond allowance

Geometry The prescribed metric, extension, holonomy, locations, time calibration, path certificates, and any independent metric-update law

bottomrule

Geometry is not folded into leakage, and a classical heat-meter label is not coherent reservoir access.

Unified finite release and the five open gravitational obligations

Theorem. Fix the prescribed host of Section , a finite code, a typed complete register interface, and explicit permitted schedules. When a field interpretation is requested, impose Assumption . Then:

• calibrated destructive clocks have the complete identity ; their consumption does not consume the geometric time axis;

• complete isometries preserve the code globally, and exact whole-code recovery through the declared accessible side is equivalent to constancy of the complete complement;

• every complete classical-only accessible channel has sharp recovery obstruction , with robustness ; its cumulative-record entropy is nondecreasing;

• the conserving single-excitation schedule has the corrected recovery, leakage, snapshot, and complete-history formulas of Section ;

• under the additional one-unitary Haar hypothesis, nested coherent and destructive tensor release have the exact means , and one sampled device has the all-prefix control ;

• when an additive conserved sector is actually declared, the correct means are the direct-sum formulas –;

• entropy histories and selected thermal observations do not certify whole-code recovery, as shown by Theorems and and Proposition ;

• fixed tensor-release capacities obey the sum constraint. The inverse-square rounded proxy and its cutoff have the conditional properties of Section ;

• physical entropy comparison and quantum recovery have the separate allowances in Theorems and ;

• thermal reset requires the independent implementation hypotheses of Theorem . Host placement preserves these finite statements only through the written access-preserving identifications. It does not supply their missing physical acquisition, dynamical, or endpoint identifications.

Proof. Items (i)–(iii) are Theorems , , , and . Item (iv) is Theorems and . Items (v)–(vi) are Theorem , Proposition , and Theorem . The cited separation constructions prove (vii). Theorems and and Proposition prove (viii). The named comparison theorems prove (ix), and Theorem proves (x). Proposition specifies the host-identification step. No continuum acquisition or geometry-update channel is among the constructed maps.

What remains open

The five gravitational obligations retained from the Hawk05-B plan are not discharged by Theorem .

begingroup 1.15 toprule Obligation Declared host-side contribution Still unconstructed here

midrule endfirsthead toprule Obligation Declared host-side contribution Still unconstructed here

midrule endhead bottomrule endfoot

Microscopic complete state and dynamics A prescribed free scalar or compatible free Maxwell model, its extension and state, and a finite packet encoding Interacting gravitational evaporation, backreaction, a dynamical horizon, and Einstein evolution

Typicality in conserved sectors A specified holonomy, and, only if separately supplied, an additive charge decomposition with the exact direct-sum calculation A derivation of Haar or conserved-sector typicality from the cone or its vacuum current

Emission clock and capacity history Calibrated finite clocks, complete composition bounds, causal routing, and a supplied compatible capacity schedule A capacity history derived from a gravitational luminosity or metric, with justified temporal coarse graining

Coherent receiver if recovery is claimed A mathematically returned quantum register and a specified decoder; classical records may accompany it A physical radiation-to-memory acquisition channel with complete error, locality, storage, and control allowances

Same-model endpoint comparison The E1–E4 checklist, complete-complement/Choi bounds, retaining-memory witness, and cutoff/recurrence distinctions A gravitational endpoint, a selected remnant or final burst, and a proved access-preserving endpoint comparison

endgroup

A free-field propagator, finite propagation, or a renormalized one-point function supplies only its own declared data. A thermal spectrum is not a complete accessible algebra. The cone apex is not a horizon.

Potential significance and the limit of novelty
The useful combined certificate is

declared geometry and field domains + complete finite preparation and access
+two explicit return types + reference-preserving error allowances
+separate entropy, recovery, and thermal questions.

It permits an informative classical experiment to be analyzed without promoting its record to coherent recovery, or a finite entropy tent to gravitational evaporation.

The contribution is this integrated proof package and its corrections, not the invention of entanglement breaking, exact correction, Page–Sen averaging, Dirichlet statistics, or Landauer balance. No general priority claim is made for the component formulas. A physical successor must construct the five missing obligations in one compatible model rather than infer them from the completed finite proxy.

Standing programme boundaries
The finite assembly neither repairs the separate characteristic quantum-product programme nor supplies its inherited full characteristic map. In the supplied programme terminology:

$$\text{boxed}\,\mathrm{O}.1 = \text{texttt}\,\mathit{NOT_CLOSED}.$$

XVI-N and its undischarged Route B are not discharged. XV-B's downstream admission exit is not opened. The historical H2 boundary is not confused with XIV-I's narrower constructed I2 internal amplitude; the full joint maps at I3 and J2 are not supplied [CheungStops]. No inherited characteristic quantum coefficient is admitted.

No Einstein dynamics, physical spin-2 Hessian, gravitational Page curve, holographic dictionary, or spacetime emergence is constructed. These are scope limits, not universal nonexistence theorems.

appendix

A self-contained evaluation of the Page–Sen mean

This appendix evaluates the standard formula [PageAverage,Sen]. Natural logarithms are used until the last step. Let $1 \le m \le n$, $D = mn$, and let G be an $m \times n$ matrix of independent standard complex Gaussian entries. Put

$$W = GG^{\dagger}, \qquad T = \operatorname{Tr} W, \qquad \rho = W/T.$$

Gaussian radial decomposition shows that T is Gamma distributed with shape D, independent of ρ. Consequently

$$\mathbb{E}[T\ln T] = D\psi(D+1),$$

where ψ is the logarithmic derivative of Γ. Since $W = T\rho$,

$$\mathbb{E}S_{\mathrm{n}}(\rho) = \psi(D+1) - \frac{1}{D}\mathbb{E}\operatorname{Tr}(W\ln W).$$

Write $a = n - m$. The unordered eigenvalue density of W is proportional to

$$e^{-\sum_i x_i} \prod_i x_i^a \prod_{i<j} (x_i - x_j)^2, \qquad x_i > 0.$$

This follows from singular-value decomposition of the Gaussian matrix. Expanding the Vandermonde determinant in orthogonal polynomials and integrating all but one variable gives the one-point density

$$R_1(x) = x^a e^{-x} \sum_{j=0}^{m-1} \frac{[L_j^a(x)]^2}{h_j}, \qquad h_j = \frac{\Gamma(j + a + 1)}{j!}.$$

Here *L_j^a* are generalized Laguerre polynomials, normalized by

$$\int_0^\infty x^a e^{-x} L_j^a(x) L_\ell^a(x)\, \mathrm{d}x = h_j \delta_{j\ell}.$$

The determinant expansion yields $\int R_1 = m$, so the normalization in is fixed.

The recurrence

$$x L_j^a = (2j + a + 1) L_j^a - (j + 1) L_{j+1}^a - (j + a) L_{j-1}^a$$

implies

$$A_j := \int_0^\infty x^{a+1} e^{-x} [L_j^a(x)]^2 \, \mathrm{d}x = (2j + a + 1) h_j.$$

The finite-series definition of *L_j^a*, differentiated in a, gives

$$\partial_a L_j^a = \sum_{\ell=0}^{j-1} \frac{L_\ell^a}{j - \ell}.$$

Only the $\ell = j - 1$ term contributes when this is paired with *xL_j^a*. Orthogonality therefore gives

$$\int_0^\infty x^{a+1} e^{-x} L_j^a \, \partial_a L_j^a \, \mathrm{d}x = -j h_j.$$

Differentiate $A_j = (2j + a + 1) h_j$. Differentiation under the integral is justified for $a > -1$ by local integrability at zero and exponential decay at infinity. One obtains

$$\frac{1}{h_j} \int_0^\infty x^{a+1} e^{-x} \ln x [L_j^a(x)]^2 \, \mathrm{d}x = (2j + a + 1) \psi(j + a + 1) + 1 + 2j.$$

The elementary finite sums are

$$\sum_{j=0}^{m-1} (2j + a + 1) = mn, \qquad \sum_{j=0}^{m-1} (1 + 2j) = m^2.$$

Using

$$\psi(j + a + 1) = \psi(n + 1) - \sum_{\ell=j+a+1}^{n} \frac{1}{\ell}$$

and exchanging finite sums gives

$$\sum_{j=0}^{m-1} (2j + a + 1) \psi(j + a + 1) = mn \psi(n + 1) - \frac{m(m + 1)}{2}.$$

Substitution into – yields

$$\mathbb{E}\,\mathrm{Tr}\,(W \ln W) = mn \psi(n + 1) + \frac{m(m - 1)}{2}.$$

Equation becomes

$$\mathbb{E} S_{\mathrm{n}}(\rho) = \psi(mn + 1) - \psi(n + 1) - \frac{m - 1}{2n}.$$

Finally $\psi(\ell + 1) = \mathrm{Harm}\, \ell - \gamma_E$, and division by $\ln 2$ proves , including $m = 1$.

The Haar fourth moment for the subcode allowance

Haar invariance permits $J|i\rangle$ to be the first d coordinate vectors. Write

$$V_{ab,i} = U_{(a,b),i}, \qquad 1 \le a \le r, \quad 1 \le b \le b.$$

For $D = rb \ge 2$, the order-two Haar identity is

$$\begin{aligned}\mathbb{E}[U_{xi}U_{zk}\overline{U_{yj}U_{w\ell}}] &= \frac{\delta_{xy}\delta_{zw}\delta_{ij}\delta_{k\ell} + \delta_{xw}\delta_{zy}\delta_{i\ell}\delta_{kj}}{D^2-1} \\ &\quad - \frac{\delta_{xy}\delta_{zw}\delta_{i\ell}\delta_{kj} + \delta_{xw}\delta_{zy}\delta_{ij}\delta_{k\ell}}{D(D^2-1)}.\end{aligned}$$

It follows directly from left and right unitary invariance: only the two row and two column pairings are possible; contracting unitarity determines their coefficients. It is also the standard second-order unitary integration formula [CollinsSniady].

The complementary normalized Choi entries are

$$(\omega_{QB})_{ib,jb'} = \frac{1}{d}\sum_a V_{ab,i}\overline{V_{ab',j}}.$$

Expanding *Tr ω_QB^2* and applying gives four index counts

$$dr^2b, \qquad d^2rb^2, \qquad d^2r^2b, \qquad drb^2.$$

The last two have the additional negative factor *1/D*. Thus

$$\mathbb{E}\operatorname{Tr}\omega_{QB}^2 = \frac{r(D/d-1) + b(D-1/d)}{D^2-1}.$$

Similarly,

$$(\sigma_B)_{b,b'} = \frac{1}{d}\sum_{a,i} V_{ab,i}\overline{V_{ab',i}},$$

and its four counts are

$$d^2r^2b, \qquad drb^2, \qquad dr^2b, \qquad d^2rb^2.$$

Consequently

$$\mathbb{E}\operatorname{Tr}\sigma_B^2 = \frac{r(D-1/d) + b(D/d-1)}{D^2-1}.$$

Since *Tr_Q ω_QB = σ_B*,

$$\|\omega_{QB} - I_Q/d \otimes \sigma_B\|_2^2 = \operatorname{Tr}\omega_{QB}^2 - \frac{1}{d}\operatorname{Tr}\sigma_B^2.$$

Subtracting the preceding expectations gives

$$\mathbb{E}\|\omega_{QB} - I_Q/d \otimes \sigma_B\|_2^2 = \frac{(1-d^{-2})(bD-r)}{D^2-1} = \frac{(1-d^{-2})r(b^2-1)}{D^2-1}.$$

This is . In particular it vanishes at *b = 1*, as required for a completely released finite code.

Independent analytic boundaries and documentary corrections

Descent is not supplied by finite readout

For completeness, the elementary reconstruction constraint used in the documentary characteristic-map discussion is independent of the finite channel calculations.

Proposition. Let *Rec: P → K* be surjective and *M: P → Y* linear. A unique linear *M̃: K → Y* with *M = M̃Rec* exists if and only if

$$\ker\mathrm{Rec} \subseteq \ker M.$$

Proof. A factorization annihilates every reconstruction-null presentation. Conversely define $\bar{M}(\mathrm{Rec}\, p) = M(p)$. The kernel inclusion makes this independent of the chosen presentation; surjectivity gives uniqueness.

For a normal coordinate q,

$$q\delta'(q) = -\delta(q).$$

Hence a presentation whose two terms reconstruct to $q\delta^{\{'\}} \otimes R$ and $\delta \otimes R$ can be null even when each term is nonzero. A highest presentation term is not automatically an intrinsic joint symbol. This observation supplies no full characteristic map, joint section, transverse estimates, contact compatibility, or output wavefront theorem for the separately inherited graph.

Documentary references that are not technical inputs

The supplied component plan names VII-J-ST and X-J-ST-2. Their role here is documentary; the locality and non-acquisition statements actually used have been written explicitly. No unavailable theorem is promoted to an inspected proof.

The supplied description of Oai282-A concerns local Ward improvement and coverage [CheungOai282]. It is not used as a theorem deriving Unruh radiation. The supplied Oai227-A description concerns one two-spin finite comparison [CheungOai227]; general Choi bounds used here are proved in Lemma . These corrections prevent unrelated catalogue comparisons from becoming generic physical premises.

The I-B-VP continuum claims are retained as field-input descriptions. No missing numerical coefficient, detector coupling, or renormalization identity is invented. Exact versions of those inputs must be frozen when a specific physical packet is proposed.

Declarations

Nature of the work. This is an analytical finite-channel study with a declared, prescribed-background field interface. No empirical dataset is used. The proofs concern the objects defined in this manuscript or the explicitly named standard external approximation inputs.

Attribution and novelty. The exact correction criterion, entanglement-breaking mechanism, Page–Sen average, Haar integration, entropy continuity, Petz construction, and thermal identity have established sources. The contribution claimed here is their scoped integration, the local proof package, explicit error accounting, and correction of the identified inference errors. No blanket priority claim or independent verification of the registered corpus is asserted.

Authorship and bibliographic metadata. The new manuscript's author metadata are intentionally not inferred. Hawk05-A and Hawk05-B are cited without resolving inconsistent supplied author-field descriptions. Internal manuscript dates, acceptance, registration, and journal publication are not inferred from the separately cited Enter journal article.

Execution and submission status. The source uses an internal bibliography and requires no external figures or datasets. It is written for pdfLaTeX. No successful compilation, proof-assistant verification, numerical execution, independent symbolic rerun, laboratory realization, referee approval, or acceptance of Hawk05-ST is asserted. Author metadata, compilation, reference availability, and independent technical review remain submission checks rather than claimed events.

Physical and programme scope. No gravitational evaporation channel, Page curve of gravitational evaporation, Einstein dynamics, spacetime emergence, physical spin-2 Hessian, or inherited characteristic quantum coefficient is constructed. O.1 stays NOT_CLOSED. XVI-N and Route B are not discharged. XV-B's downstream admission exit is not opened. The existence of technical manuscripts bearing those identifiers is not denied.

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