# Conditional Page-Curve Extrapolations under Enter

*Compatible capacities, conserved-sector proxies, two return types, and cutoff certificates*

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*15 October 2026*

**Abstract**

We develop conditional Page-curve extrapolations from a finite Haar proxy while retaining the access and output distinctions of the Philosophy of Enter. Nonuniform coherent cell release gives the Page–Sen mean, whereas destructive readout gives an append-only classical record with a different exact entropy mean. Their entropy difference is basis-dependent relative-entropy coherence, not a whole-code recovery error or a thermodynamic cost. A compatibility theorem identifies a missing condition in a proposed clock replacement: a fixed-dimensional tensor-release proxy requires the sum of its two logarithmic capacities to remain constant. Consequently, identifying radiation capacity with a coarse entropy production law larger than black-hole entropy loss is not merely a reparametrization of Hawk05-B. We give an explicit rounded, fixed-dimensional realization for the compatible case and an exact direct-sum calculation for a conserved additive-charge sector. A positive-mass inverse-square clock yields a conditional capacity crossover, its cutoff criterion, and a peak-location stability bound. Physical entropy transfer, ensemble typicality, and quantum recovery receive separate quantitative allowances. A proposed Petz decoder is made into a deterministic channel on its entire input space and can be certified by its decoded Choi discrepancy. No occupation law, entropy slope, restricted observable algebra, or finite termination supplies the missing physical comparisons. The result is a proved finite extrapolation framework and a collection of explicitly conditional evaporation sketches, not a microscopic gravitational Page curve, an island construction, or spacetime emergence.

**Bibliographic record**

| | |
|---|---|
| **Title** | Conditional Page-Curve Extrapolations under Enter |
| **Author** | Cheung, Ngo |
| **Publication date** | 2026/10/15 |
| **Institution** | Enter |
| **Report number** | Hawk05-C |
| **Publisher** | Cheung Ngo Medical Limited |
| **First page** | 1 |
| **Last page** | 23 |
| **Language** | en |

**Mathematical scope.** All channel and random-state results below are finite dimensional. The semiclassical clock is an additional supplied model. The physical channel, receiver, and typicality allowances remain unfilled until justified in one compatible microscopic theory. The manuscript does not replace an unconstructed gravitational interface by a finite proxy and then declare the identification proved.

## Enter background, provenance, and the corrected extrapolation

### The Philosophy of Enter

Cheung's Philosophy of Enter distinguishes destructive readout, quantum return, physical implementation, and analogy. Enter consumes a measured quantum name at its interface and returns a classical record. A decoder instead acts on declared accessible systems and returns a quantum target; whole-code correctness includes untouched reference correlations. Consumption is not physical annihilation, and an input–output channel does not determine a thermal reset machine [CheungPhilosophy].

The present paper adopts ordinary finite Born instruments explicitly. Cheung's operational characterization applies inside its stated strategy class; it is not used to derive a gravitational field instrument or enlarge that class [CheungEnter,CheungI]. The four questions guiding every extrapolation are:

boxed What is prepared? What evolves? What is accessible? What is returned?

Hawk05-A separates classical accumulation from quantum recovery. Hawk05-B then constructs a finite coherent Haar history with the Page–Sen mean and a different exact classical-record mean [Hawk05A,Hawk05B]. We retain those objects as proxies, not gravitational identifications.

**Three levels of claim**

We distinguish:

• a finite channel theorem;

• a conditional identification and its quantitative allowance;

• a physical conclusion after the identification has been proved. This paper proves statements at level (i), proves implications at level (ii), and does not assert that the gravitational hypotheses required for level (iii) have been discharged.

The Page–Sen average is a random-state theorem [PageAverage,Sen]. A black-hole time profile requires additional entropy and luminosity assumptions, as in Page's time-dependent analysis [PageTime]. Island and entanglement-wedge calculations have further geometric and dynamical hypotheses [Penington,IslandReview]. None is instantiated by a finite capacity plot.

**Debugging the proposed extrapolations**

begingroup 1.12 Repairs required before extrapolating the finite proxy.

toprule Proposed inference Corrected statement

midrule endfirsthead toprule Proposed inference Corrected statement

midrule endhead bottomrule endfoot Choose arbitrary continuous $r(t)$, $b(t)$ by exponentiating two entropies. Finite dimensions are integers. In a fixed tensor-release proxy, $r(t)b(t) = D$; logarithmic capacities must therefore have constant sum, up to declared rounding error.

Clock replacement has only a channel-continuity error. The budget also includes finite Page deficit, capacity rounding, ensemble mismatch, and temporal compatibility. A collection of snapshot formulas is not automatically one emission history.

A greybody occupation fixes a cell dimension. An occupation is a state statistic. Turning it into a dimension, probability, cutoff, or entropy-rate weight requires a specified additional map and truncation convention.

The record–quantum entropy gap measures all consumed information. For a specified basis it equals relative-entropy coherence of one accessible marginal. It is state- and basis-dependent and is not whole-code recovery loss.

Restricting access produces a Page tent. Restriction defines an observable task. A tent additionally requires a state ensemble and a compatible capacity history. A product state has zero entropy on every tensor prefix.

Two commuting copies of a matrix algebra cannot exist. They can exist on independent tensor factors. What is obstructed is two commuting faithful representations that both act as the same unknown logical algebra on the encoded code.

A conserved sector remains an ordinary tensor product. A fixed total additive charge generally decomposes into a direct sum of charge-compatible tensor blocks. Its entropy includes charge weight entropy; the unrestricted Page formula cannot be applied with only two aggregate dimensions.

A falling radiation entropy implies a constant complement. A pure input's output entropy and the complete complement's dependence on all code inputs are different questions. Exact recovery may coexist with a mixed fixed syndrome.

A positive-cutoff plot necessarily includes a Page turnover. It includes a turnover only if the declared capacity crossover is reached before the cutoff. The clock can stop on the increasing branch.

A named Petz map establishes physical recovery. It must be a well-defined deterministic channel on the actual access interface, have a proved task error, and have a certified implementation. An algebraic target may be narrower than a whole code.

endgroup

The proposal's unspecified ``2026 algebraic reading'' is not treated as an established universal statement or attributed to an unidentified recent paper. Section instead states a finite access theorem and separately identifies an external asymptotic proposal with its own hypotheses.

## Complete interfaces and the two return types

Let $C$ be a variable code with $\dim C = d \geq 2$, and let $Q$ be an untouched reference.

**Definition.** A complete interface is an isometry

$$V : C \to R \otimes B,$$

with accessible channel and complete complement

$$\mathcal{N}(X) = \mathrm{Tr}_B(VXV^\dagger), \qquad \mathcal{C}(X) = \mathrm{Tr}_R(VXV^\dagger).$$

Every unreturned purifier, measurement environment, lost mode, and admitted implementation factor belongs to $B$.

Use the normalizations

$$d_{\mathrm{tr}}(\rho,\sigma) = \frac{1}{2}\|\rho - \sigma\|_1, \qquad d_\diamond(\Phi,\Psi) = \frac{1}{2}\|\Phi - \Psi\|_\diamond,$$

and define

$$\begin{aligned} \mathcal{T}_\tau(X) &= \mathrm{Tr}(X)\tau, \\ \varepsilon_{\mathrm{rec}}(\mathcal{N}) &= \inf_{\mathcal{D}\,:\,R \to C\ \mathrm{CPTP}} d_\diamond(\mathcal{D}\mathcal{N}, \mathrm{id}_C), \\ \delta_{\mathrm{full}}(\mathcal{C}) &= \inf_{\tau_B \in \mathcal{S}(B)} d_\diamond(\mathcal{C}, \mathcal{T}_{\tau_B}). \end{aligned}$$

The diamond norm includes arbitrary reference extensions [Watrous]. Set

$$a_d = 1 - d^{-2}, \qquad b_d = 1 - d^{-1}.$$

All entropies are in bits unless a natural logarithm is printed explicitly.

**Lemma.** Every isometric channel has a CPTP left inverse and preserves the trace norm of every referenced operator. CPTP pre- and postprocessing contract $d_{\{\diamond\}}$. Consequently

$$|\varepsilon_{\mathrm{rec}}(\mathcal{N}) - \varepsilon_{\mathrm{rec}}(\mathcal{N}')| \mathrm{led}_\diamond(\mathcal{N}, \mathcal{N}').$$

*Proof.* For a fixed state $\omega_C$, the map

$$\mathcal{L}_V(Y) = V^\dagger Y V + \mathrm{Tr}((I - VV^\dagger)Y)\omega_C$$

is CP, trace preserving, and left-inverts $\mathrm{Ad}_V$. An isometric embedding preserves all nonzero singular values, also after reference extension. Trace-distance contraction on referenced states gives diamond contraction. Insert an arbitrary decoder in the triangle inequality, take infima, and interchange the compared channels to prove stability.

**Theorem.** An accessible output exactly recovers the whole code if and only if its complete complement is replacement:

$$\mathcal{D}\mathcal{N} = \mathrm{id}_C \quad \Leftrightarrow \quad \mathcal{C} = \mathcal{T}_{\tau_B}.$$

In that case there are a state $\sigma_K$ and an isometry $T: C \otimes K \to R$ such that

$$\mathcal{N}(X) = T(X \otimes \sigma_K)T^\dagger.$$

One may take $\dim K = \mathrm{rank}\,\tau_B$. For a pure logical input,

$$S(\mathcal{N}(\mathrm{kb}\,\psi\psi)) = S(\sigma_K).$$

Thus exact recovery does not force purity of the raw accessible output.

*Proof.* If the complement is constant, diagonalize $\tau_B = \sum_{\alpha} \lambda_{\alpha} kb \alpha\alpha$ on its nonzero support and write

$$V|i\rangle = \sum_{\alpha} \sqrt{\lambda_{\alpha}} |u_{i\alpha}\rangle_R |\alpha\rangle_B.$$

The complementary equations on matrix units imply $\langle u_{j\beta} \mid u_{i\alpha}\rangle = \delta_{ij}\delta_{\alpha\beta}$. They define $T(|i\rangle|\alpha\rangle) = |u_{i\alpha}\rangle$. Compression by $T^{\dagger}$, tracing $K$, and preparing a fixed state on the unused accessible subspace gives a deterministic decoder.

Conversely, dilate an exact decoder by $W: R \to CG$. The composite $(W \otimes I_B)V$ dilates the identity channel. Its Choi operator has rank one, so each Kraus operator is a scalar multiple of $I_C$. Hence the composite is $|\psi\rangle \mapsto |\psi\rangle_C|\chi\rangle_{GB}$ for fixed $\chi$. Tracing $CG$ proves complement constancy. The entropy statement follows from .

This is the standard exact correction criterion [KnillLaflamme,CheungEm05B], not a new theorem supplied merely by the Enter terminology.

**Lemma.** A replacement accessible channel has optimal recovery error $a_d$; every channel has recovery error at most $a_d$. If $d_{\diamond}(\mathcal{N}, \mathcal{T}_{\omega}) \le \eta$, then

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}) \ge \max\{0, a_d - \eta\}.$$

For a complete interface, the information–disturbance theorem gives

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}) \le \min\{a_d, \sqrt{2\delta_{\mathrm{full}}(\mathscr{C})}\}.$$

*Proof.* On a Bell input, a decoded replacement produces $I_Q/d \otimes \sigma_C$, with Bell overlap $d^{-2}$. Preparing $I_C/d$ attains the lower bound: depolarization is the uniform average of the $d^2$ Weyl conjugations, one of which is identity. This also gives the universal constant-decoder cap. The accessible-replacement lower bound follows by contraction and the reverse triangle inequality.

For the last assertion use the finite information–disturbance inequality

$$\frac{1}{4}\Big(\inf_{\mathscr{D}} \|\mathscr{D}\mathcal{N} - \mathrm{id}_C\|_{\diamond}\Big)^2 \le \inf_{\tau} \|\mathscr{C} - \mathscr{T}_{\tau}\|_{\diamond}$$

and substitute the half-diamond normalization [KSW].

### Destructive Enter

For a basis $\{|y\rangle\}$ of a measured factor $A$, define

$$(\mathrm{Enter}_A \otimes \mathrm{id}_K)(X) = \sum_y \mathrm{kb}\, yy_Y \otimes (\langle y|_A \otimes I_K) X (|y\rangle_A \otimes I_K).$$

The measured name $A$ is not returned. Its complete dilation can be

$$|y\rangle_A \mathrm{longmapsto}\, |y\rangle_Y |y\rangle_L,$$

with $L$ unreturned. A continuation with joint access to $YL$ would have a different quantum interface.

**Proposition.** For every purely classical output channel on a $d$-dimensional code,

$$\varepsilon_{\mathrm{rec}}(\mathscr{M}) \ge b_d.$$

Computational-basis measurement attains the bound. For an unedited append-only record,

$$H(Y_{\le k+1}) = H(Y_{\le k}) + H(Y_{k+1} \mid Y_{\le k}) \ge H(Y_{\le k}).$$

Moreover,

$$\mathrm{dropBit}_Y \circ \mathrm{Enter}_A = \mathrm{Tr}_A$$

on all referenced operators.

*Proof.* Any decoder following a classical-output channel is measure-and-prepare, so its Bell output is separable [HorodeckiShorRuskai]. Every separable state has Bell overlap at most $1/d$, since a product state has overlap $\mathrm{Tr}(\rho_Q^{T}\sigma_C)/d \le 1/d$. The Bell-projector test gives error at least $b_d$. Basis-state preparation gives dephasing, the average of $d$ phase conjugations, and has matching error $b_d$.

The entropy assertion is the classical chain rule. For the last assertion, expand $X = \sum_{y,z} X_{yz} \otimes kb\, yz_A$.

Readout followed by discard returns ∑_*y X_yy*, the partial trace. The blocks may include any reference.

In particular, a record-only endpoint cannot have a constant complete complement. Terminal residual readout and discard do not improve the already-returned accessible channel [CheungEn07A,CheungEn07B].

## Nonuniform cells: an exact temporal proxy

Fix cell dimensions *s_1*, …, *s_N* ≥ *2* and a final core dimension *b_{*} ≥ 1*. Let

$$\mathscr{H}_{\mathrm{int}} = \left(\mathrm{bigotimes}_{j=1}^{N} \mathbb{C}^{s_j}\right) \otimes \mathbb{C}^{b_*}, \qquad D = b_* \prod_{j=1}^{N} s_j.$$

Prepare a fixed isometry *J: C → ℋ_int*, choose one logical-input-independent unitary *U ∈ U(D)*, and then release the cells in order. Earlier outputs are untouched.

At cut *k*,

$$r_k = \prod_{j=1}^{k} s_j, \qquad b_k = b_* \prod_{j=k+1}^{N} s_j, \qquad r_k b_k = D.$$

Haar statements refer to one Haar sample of *U*, not independently sampled states at different cuts. The pure input is fixed independently of *U*; *U* is known when a decoder is invoked.

A finite conserving implementation may give each occupied memory and outgoing cell a common gap *ω*, with a vacant zero-energy sector. The preparation occupies all memory cells. Extend *U* on the all-occupied, degenerate internal sector and transfer each cell by SWAP. These operations commute with the declared sum of occupied-sector energies. This establishes a register conservation law, not a gravitational mass law [Hawk05B,CheungHawk02B].

For integers *r, b ≥ 1*, put

$$m = \min(r, b), \quad n = \max(r, b), \quad D = rb, \qquad \mathrm{H}_\ell = \sum_{j=1}^{\ell} \frac{1}{j},$$

and define

$$\mathrm{P}(r, b) = \frac{\mathrm{H}_{rb} - \mathrm{Harm}\, n - (m-1)/(2n)}{\ln 2},$$

$$\mathrm{L}(r, b) = \frac{\mathrm{H}_{rb} - \mathrm{Harm}\, b}{\ln 2}.$$

**Lemma.** A Haar pure state on ℂ^*r* ⊗ ℂ^*b* has mean reduction entropy *P(r, b)*.

*Proof.* The Gaussian–Wishart evaluation in Appendix gives . This is the established Page–Sen formula [PageAverage,Sen].

**Theorem.** For every fixed pure input independent of *U*, coherent radiation and immediate destructive cell readout have the exact means

$$\mathbb{E}_U S(R_k) = \mathrm{P}(r_k, b_k), \qquad \mathbb{E}_U H(Y_{\le k}) = \mathrm{L}(r_k, b_k).$$

The classical mean increases strictly at each release. The coherent mean increases on cuts with *r_k ≤ b_k* that remain on that side, and decreases on cuts with *r_k ≥ b_k* that remain on that side. If the capacity crossing is traversed, a maximal cut is among the two cuts bracketing it.

If *b_{*} = 1*, the coherent endpoint mean is zero while the record endpoint mean is

$$\frac{\mathrm{Harm}\, D - 1}{\ln 2} > 0.$$

If the schedule stops before a crossing, no turnover is guaranteed.

*Proof.* Haar invariance makes *UJ|ψ⟩* a Haar unit vector, and Lemma applies at every cut.

Write its product-basis components as normalized independent complex Gaussians. A radiation prefix has *b_k* squared Gaussian components. The *r_k* prefix probabilities therefore have a Dirichlet distribution with all parameters *b_k*. Its moment identity is

$$\mathbb{E}\sum_{y} p_y^t = r_k \frac{\Gamma(b_k + t)\Gamma(D)}{\Gamma(b_k)\Gamma(D + t)}.$$

Differentiate at *t = 1*. Using *ψ_0(ℓ + 1) = H_{ℓ} − γ_E* gives the record mean. Later SWAPs act on unmeasured factors, so immediate readout has precisely these prefix probabilities.

For a same-side coherent step, write the smaller and larger dimensions as *m, n*, and transfer an integer factor *s ≥ 2* so that the next early-side dimensions are *sm, n/s*. Remaining on that side implies *n ≥ s^2m*. The Page-mean increment in natural units is

$$\sum_{j = n/s + 1}^{n} \frac{1}{j} - \frac{(s^2 - 1)m - (s - 1)}{2n} \geq 1 - \frac{1}{s} - \frac{s^2 - 1}{2s^2} = \frac{(s - 1)^2}{2s^2} > 0.$$

Exchange the two factors for the late side. The record increment is *(H_{b_{k − 1}} − H_b_k)/ln 2 > 0*. The endpoint assertions follow by substitution.

A large cell can jump over the equal-capacity point. The precise finite maximum need not be located at a time satisfying *r_k = b_k*, because that equality may occur at no cut.

**Finite deficits and single-history control**

**Proposition.** Let *T(r, b) = log_2 m*. Then

$$0 \leq T(r, b) - \mathrm{P}(r, b) \leq u(r, b) := \log_2 (1 + m/n),$$

$$0 \leq \log_2 r - \mathrm{L}(r, b) \leq \frac{1}{b \ln 2}.$$

For a single sampled Haar history and every *z > 0*,

$$\mathbb{P}\left\{ \max_k \left(T(r_k, b_k) - S(R_k)\right) > z \right\} \leq \min\left\{ 1, \frac{\sum_k u(r_k, b_k)}{z} \right\}.$$

No independence of cuts is assumed.

*Proof.* Haar purity is *𝔼Tr ρ_m^2 = (m + n)/(mn + 1)*, obtained by the two-copy swap test on *𝔼kb vv^{⊗2} = (I + F)/(D(D + 1))*. Since *S(ρ) ≥ −log_2 Tr ρ^2*, Jensen's inequality gives

$$\log_2 m - \mathbb{E}S(\rho) \leq \log_2 \left( \frac{m(m + n)}{mn + 1} \right) \leq u(r, b).$$

The upper entropy bound is the rank bound.

The harmonic sum obeys

$$\ln \frac{D + 1}{b + 1} \leq \mathrm{Harm}\, D - \mathrm{Harm}\, b \leq \ln \frac{D}{b}.$$

Its difference from *ln r* is at most *ln (1 + 1/b) ≤ 1/b*, proving . Finally, every capacity deficit is nonnegative, and its expectation is bounded by . Apply Markov's inequality to their sum, which dominates their maximum.

**What the entropy gap actually means**

Let *Δ_R* dephase the declared radiation basis. Its diagonal probabilities equal the destructive prefix record probabilities.

**Proposition.** For every state *ρ_R*,

$$H(Y) - S(\rho_R) = S(\Delta_R \rho_R) - S(\rho_R) = D_2(\rho_R \| \Delta_R \rho_R) \geq 0.$$

For the Haar proxy,

$$\mathrm{L}(r, b) - \mathrm{P}(r, b) = \begin{cases} \dfrac{r - 1}{2b \ln 2}, & r \leq b, \\ \dfrac{\mathrm{Harm}\, r - \mathrm{Harm}\, b + (b - 1)/(2r)}{\ln 2}, & r \geq b. \end{cases}$$

This is not a whole-code recovery diagnostic.

*Proof.* The diagonal logarithm satisfies *Tr (ρ_Rlog_2 Δ_Rρ_R) = Tr (Δ_Rρ_Rlog_2 Δ_Rρ_R)*. Positivity ensures the required support containment. The relative-entropy identity and nonnegativity follow. Subtract from for the

two cases.

For a Bell input to a *d*-dimensional identity channel, the accessible marginal is *I/d*, so the marginal gap is zero. Nevertheless, destructive basis readout has recovery error *b_d*. Thus a vanishing gap does not mean no quantum-recovery loss.

The quantity is the relative entropy of coherence in the selected basis [Baumgratz]. It is not a basis-independent count of information destroyed, a residual leakage, or a Landauer cost.

## Typicality and physical transfer require different allowances

### A precise ensemble comparison

Let $\mu$ be a distribution of fixed unitaries on the same internal space. Define

$$d_{\mathrm{TV}}(\mu, \mu_{\mathrm{Haar}}) = \sup_{E} |\mu(E) - \mu_{\mathrm{Haar}}(E)|.$$

**Proposition.** If *d_TV(μ, μ_Haar)* ≤ τ, then at a fixed cut

$$|\mathbb{E}_{\mu} S(R) - \mathrm{P}(r, b)| \le \tau \log_2 m,$$

$$|\mathbb{E}_{\mu} H(Y) - \mathrm{L}(r, b)| \le \tau \log_2 r.$$

These statements apply to fixed inputs independent of the sampled unitary.

*Proof.* Entropy ranges are *0 ≤ S(R) ≤ log_2 m* for a pure complete proxy and *0 ≤ H(Y) ≤ log_2 r*. The expectation difference of a function in *[0, L]* is at most *L d_TV*, by integration of its level sets.

Total variation is a strong sufficient assumption, not a claim that physical scrambling achieves it.

A weaker hypothesis suffices for a leading capacity law. For a fixed pure input, let

$$M_2^{\mu} = \mathbb{E}_{\mu} \mathrm{kb}\, vv^{\otimes 2}, \qquad \left\| M_2^{\mu} - \frac{I + \mathrm{F}}{D(D+1)} \right\|_1 \le \chi.$$

The purity swap witness has operator norm one, so

$$\mathbb{E}_{\mu} \mathrm{Tr}\, \rho_m^2 \le \frac{m+n}{mn+1} + \chi.$$

The proof of Proposition then gives

$$\mathbb{E}_{\mu}(\log_2 m - S(R)) \le \min\left\{ \log_2 m, \log_2 \left[ m\left( \frac{m+n}{mn+1} + \chi \right) \right] \right\}.$$

Two-copy information can control this leading deficit; it does not by itself establish either exact entropy mean.

**Remark.** If the sampled unitary is forgotten and the accessible channel itself is Haar-averaged, then

$$\mathbb{E}_{U} \mathcal{N}^{U}(X) = \mathrm{Tr}\,(X) \frac{I_R}{r}.$$

That channel is replacement and has recovery error *a_d*. The Page average is *𝔼_US(ρ_R^U)*, not *S(𝔼_Uρ_R^U)*. Likewise, the expression ``the complement is diamond-close to Haar" has no meaning until a comparison channel, ensemble, input quantifiers, and treatment of the random control are specified.

### Finite-dimensional entropy continuity

For an integer *L ≥ 1*, define

$$c_L(\eta) = \begin{cases} 0, & L = 1, \\ h_2(t) + t \log_2 (L-1), & L \ge 2, \quad t = \min\{\eta, 1 - 1/L\}. \end{cases}$$

If states on an *L*-dimensional space have trace distance at most *η*, then their entropy difference is at most *c_L(η)* [Audenaert].

**Definition.** At a declared cut, supply:

• a physical accessible channel and a proxy accessible channel on the same code, after justified access-preserving identifications;

• a common accessible comparison dimension *L*, with *d_{◇}(𝒩_phys^U, 𝒩_proxy^U) ≤ η*;

• a physical detector channel compared with the proxy basis readout, with channel allowance $v$, and common record dimension $K$;

• a distribution $\mu$ of proxy unitaries with TV allowance $\tau$, if an exact Haar-mean comparison is desired;

• real target capacities $A$, $B$ and integer proxy dimensions $r$, $b$ with

$$|\log_2 r - A| \le e_R, \qquad |\log_2 b - B| \le e_B.$$

The physical and proxy families must be coupled on these interfaces. If a temporal claim is made, the cuts must belong to compatible complete histories.

**Theorem.** Under Definition , for the declared fixed pure input,

$$|\mathbb{E}S(R_{\mathrm{phys}}) - \min(A,B)| \le c_L(\eta) + \tau \log_2 m + u(r,b) + \max(e_R, e_B),$$

$$|\mathbb{E}H(Y_{\mathrm{phys}}) - A| \le c_K(\min\{1, \eta + v\}) + \tau \log_2 r + \frac{1}{b \ln 2} + e_R.$$

The record-only recovery obstruction remains independent of these entropy allowances.

*Proof.* The channel comparison bounds the compared output trace distances, including references, by $\eta$. Detector composition contraction and the triangle inequality give record distance at most $\min\{1, \eta + v\}$. Apply the finite entropy continuity bounds and take expectations. Proposition replaces the proxy ensemble mean by its Haar mean. Proposition compares those means with $\log_2 m$ and $\log_2 r$. Finally, $\min$ is one-Lipschitz in the maximum norm:

$$|\min(\log_2 r, \log_2 b) - \min(A,B)| \le \max(e_R, e_B).$$

This proves both budgets. The classical recovery lower bound is Proposition .

The dimensions $L$, $K$ are those of the actual common comparison spaces. If errors populate additional sectors, their dimensions cannot be deleted from the continuity modulus without a support estimate.

Compatible preparation, evolution, collection, and implementation channel errors telescope by replacing one factor at a time. This yields a sum of channel allowances, capped at one. A metric discrepancy, power error, or selected occupation error cannot be inserted into that sum without a conversion theorem [CheungHawk04D,CheungEn02C].

## Replacing the clock: compatibility, crossover, and cutoff

### The constant-capacity-sum requirement

**Theorem.** For the proxy of Section ,

$$\log_2 r_k + \log_2 b_k = \log_2 D$$

at every cut. If target capacities $A(t_k)$, $B(t_k)$ satisfy the rounding bounds of Definition , then

$$|A(t_k) + B(t_k) - \log_2 D| \le e_{R,k} + e_{B,k}.$$

In particular, for any two cuts $j$, $k$,

$$|[A+B](t_k) - [A+B](t_j)| \le e_{R,k} + e_{B,k} + e_{R,j} + e_{B,j}.$$

This is a necessary condition for that fixed-dimensional model, not for every possible evaporation model.

*Proof.* Equation gives $r_k b_k = D$. Subtract the two target capacities from the corresponding logarithms and use the triangle inequality.

Changing support sectors, adding fresh resources, or admitting additional inaccessible factors can produce other models. Such changes require a new complete history and state calculation. They do not preserve the fixed tensor-release theorem merely by keeping the word ``Haar."

### A supplied semiclassical clock

Only in this subsection and its clock examples set $G = \hbar = c = k_B = 1$. For a Schwarzschild mass label, the Bekenstein–Hawking entropy in bits is

$$B(M) = c_{\mathrm{BH}} M^2, \qquad c_{\mathrm{BH}} = \frac{4\pi}{\ln 2}.$$

This is a supplied semiclassical entropy law [Bekenstein,Hawking]. Identifying *B(M)* with *log_2 dim B_proxy* is an additional modelling declaration.

Let *M_0 > M_cut > 0*. Supply a positive continuous power law *P(M)* on *[M_cut, M_0]* and define

$$\dot{M} = -P(M), \qquad M(t_0) = M_0.$$

Its cutoff clock is the inverse of

$$t(M) - t_0 = \int_M^{M_0} \frac{du}{P(u)}.$$

Continuity and positivity make this inverse unique.

Supply a positive continuous capacity-production ratio *ξ(M)* and define the *target*, not the fine-grained entropy,

$$A(M) = \int_M^{M_0} \xi(u) B'(u)\, du.$$

The candidate envelope is

$$F(t) = \min\{A(M(t)), B(M(t))\}.$$

**Theorem.** For the supplied laws above:

• *A(M(t))* increases strictly and *B(M(t))* decreases strictly;

• there is exactly one crossover in the retained clock interval if and only if

$$A(M_{\text{cut}}) \geq B(M_{\text{cut}});$$

• a strict interior turnover occurs if and only if that inequality is strict;

• if *A(M_cut) < B(M_cut)*, the envelope is increasing throughout the interval. The crossover is a capacity-envelope time, not automatically a physical recovery transition or the exact maximum of a finite Page mean.

*Proof.* Differentiating gives

$$\frac{dA}{dt} = \xi(M) B'(M) P(M) > 0, \qquad \frac{dB}{dt} = -B'(M) P(M) < 0.$$

The difference $A - B$ starts at $-B(M_0)$, increases strictly in time, and is continuous. The intermediate value theorem proves existence and uniqueness exactly under the endpoint condition. The minimum selects the increasing branch before the crossing and the decreasing branch afterward.

**An inverse-square example**

Assume

$$P(M) = \frac{\lambda}{M^2}, \qquad \lambda > 0, \qquad \xi(M) = \xi > 0.$$

The flux coefficient $\lambda$ is not the surface gravity $\kappa_H$.

**Corollary.** Within the positive-cutoff interval,

$$M(t) = [M_0^3 - 3\lambda(t - t_0)]^{1/3},$$

$$t_{\text{cut}} - t_0 = \frac{M_0^3 - M_{\text{cut}}^3}{3\lambda},$$

$$A(M) = \xi c_{\text{BH}} (M_0^2 - M^2).$$

The formal crossover mass and time are

$$M_{\text{P}} = M_0 \sqrt{\frac{\xi}{1+\xi}},$$

$$t_{\text{P}} - t_0 = \frac{M_0^3}{3\lambda}\left[1 - \left(\frac{\xi}{1+\xi}\right)^{3/2}\right],$$

$$F(t_{\text{P}}) = \frac{\xi}{1+\xi} B(M_0).$$

They belong to the retained model only if *M_cut* ≤ *M_P*. No value after *t_cut* is predicted.

*Proof.* Integrate *M^2 dM = −λ dt*, then solve *A(M) = B(M)*. The cutoff condition is Theorem .

The dependence on the entropy-production ratio is the same algebra that appears in Page's semiclassical time profile [PageTime]. A half-capacity crossing is not generically half the mass or half the formal lifetime.

**Proposition.** The inverse-square clock itself can reparametrize a fixed tensor-release proxy. The assignments

$$\log_2 r = A(M), \qquad \log_2 b = B(M)$$

cannot both be exact assignments in that proxy over a nontrivial mass interval when $\xi \neq 1$. More quantitatively, they must satisfy

$$|\xi - 1|[B(M_0) - B(M)] \leq w_0 + w_M, \qquad w = e_R + e_B.$$

*Proof.* Here

$$A(M) + B(M) = \xi B(M_0) + (1 - \xi)B(M).$$

It is constant on a nontrivial interval exactly when $\xi = 1$. Subtract its initial value and apply .

Thus taking a coarse entropy-production ratio different from one does not simply ``replace the clock" in Hawk05-B. It changes the capacity model, and potentially its state ensemble.

### A compatible, rounded two-curve sketch

Choose $\xi = 1$, assume $B_0 := B(M_0) \geq 2$, and put

$$n = \lfloor B_0 \rfloor, \qquad k(t) = \min\{n, \lfloor A(M(t)) \rfloor\}.$$

Use $n$ qubit cells and one sampled Haar unitary on their $2^n$-dimensional internal space. Release a cell whenever $k(t)$ increments. This gives one nested finite history.

**Theorem.** For this declared construction,

$$r(t) = 2^{k(t)}, \qquad b(t) = 2^{n - k(t)}, \qquad D = 2^n,$$

and

$$|\log_2 r(t) - A(M(t))| \leq 1, \qquad |\log_2 b(t) - B(M(t))| \leq 1.$$

Its exact coherent and record means are

$$Q(t) = \mathrm{P}(2^{k(t)}, 2^{n - k(t)}), \qquad L(t) = \mathrm{L}(2^{k(t)}, 2^{n - k(t)}).$$

They obey

$$|Q(t) - F(t)| \leq 1 + \log_2\left(1 + 2^{-|n - 2k(t)|}\right),$$

$$|L(t) - A(M(t))| \leq 1 + \frac{2^{-(n - k(t))}}{\ln 2}.$$

If $B(M_{cut}) \geq 1$, at least one residual qubit remains at the cutoff. No terminal drain is included.

*Proof.* Before the cap in $k(t)$, the radiation rounding error is less than one and

$$n - k(t) - B(M(t)) = n - B_0 + A(M(t)) - \lfloor A(M(t)) \rfloor$$

lies between $-1$ and $1$. After the cap, both target errors are at most one because $A + B = B_0$ and $A \geq n$. Apply Proposition . If $B(M_{cut}) \geq 1$, then $A(M_{cut}) \leq B_0 - 1$, so $k(t_{cut}) \leq n - 1$.

This is a completed *proxy* calculation combining a clock with two return types. Its physical extrapolation is exactly Theorem , with $\eta(t)$, $v(t)$, $\tau$ still requiring justification.

### Clock and peak stability

**Proposition.** Suppose a supplied actual power on the same mass interval satisfies

$$(1 - \rho)\frac{\lambda}{M^2} \leq P_{\mathrm{act}}(M) \leq (1 + \rho)\frac{\lambda}{M^2}, \qquad 0 \leq \rho < 1.$$

Then its time to any mass $M$ in that interval satisfies

$$\frac{M_0^3 - M^3}{3\lambda(1+\rho)} \le t_{\mathrm{act}}(M) - t_0 \le \frac{M_0^3 - M^3}{3\lambda(1-\rho)}.$$

Separately, suppose the inverse-square envelope has an interior crossover and a function *f(t)* satisfies *sup_t |f(t) − F(t)| ≤ E*. Every attained global maximum $\hat{t}$ of *f* obeys

$$|\hat{t} - t_{\mathrm{P}}| \le \frac{2E}{v}, \qquad v = \frac{2c_{\mathrm{BH}}\lambda \min\{\xi, 1\}}{M_0}.$$

*Proof.* Integrate the bounds on *1/P_act(M)*. For the second assertion, the envelope has slope *2c_BHλξ/M* on its early branch and negative slope of magnitude *2c_BHλ/M* on its late branch. Both magnitudes are at least *v*, so

$$F(t_{\mathrm{P}}) - F(t) \ge v|t - t_{\mathrm{P}}|.$$

At a maximizing $\hat{t}$, *f(t̂) ≥ f(t_P)*, which implies *F(t_P) − F(t̂) ≤ 2E*.

The power allowance controls a clock. The entropy allowance controls a peak of a specified function. Neither supplies the other.

## Greybody weighting: what is calculable and what is declared

### Occupation and flux are not dimensions

In the stationary massless scalar setup, with the additional time–frequency counting convention, the supplied occupation is

$$n_\alpha(\omega; M) = \frac{\Gamma_\alpha(\omega; M)}{e^{8\pi M\omega} - 1}.$$

Its flux is

$$P(M) = \sum_\alpha g_\alpha \int_0^\infty \frac{d\omega}{2\pi}\, \omega n_\alpha(\omega; M).$$

Here *α* denotes the declared modes or species and *g_{α}* their counted multiplicities. The stationary field derivation and its validity hypotheses are external physical inputs [Hawking,PageEmission,CheungEm03A]. Using them at an evolving *M(t)* additionally assumes a quasistatic approximation.

If *Γ_{α}(ω; M) = γ_{α}(Mω)* for a fixed massless mode set, and the following integrals converge, then

$$P(M) = \frac{\lambda}{M^2}, \qquad \lambda = \sum_\alpha g_\alpha \int_0^\infty \frac{dx}{2\pi}\, \frac{x\gamma_\alpha(x)}{e^{8\pi x} - 1}.$$

This change of variables explains the inverse-square clock. It says nothing by itself about *r(t)* or *b(t)*.

**Proposition.** For *0 < u < 1*, a bosonic number mean *u* is compatible with both

$$(1-u)\mathrm{kb}\,00 + u\mathrm{kb}\,11$$

and the geometric state

$$\rho_u = (1-z)\sum_{n=0}^{\infty} z^n \mathrm{kb}\, nn, \qquad z = \frac{u}{1+u}.$$

They have different spectra and support dimensions. Therefore an occupation mean does not determine a Hilbert-space dimension or a Haar state ensemble.

*Proof.* Both states have mean *u*, by a direct sum of their number probabilities. The first has rank two and the second has infinite rank. Equality of one number moment does not identify their states.

Quasifreeness and covariance data can identify geometric or multimode Gaussian packet states under the additional assumptions in Cheung's En02-E and Em03-B records [CheungEn02E,CheungEm03B]. Those assumptions are stronger than a selected mean.

For a known geometric state, truncation through level *K* has tail *z^{K + 1}*. The normalized truncated marginal has trace distance *z^{K + 1}* from the original marginal. The normalized truncation of its standard pure purification instead has trace distance *√(z^{K + 1})*. These follow from the orthogonal tail and pure-state overlap. They are

state-specific estimates, not a uniform whole-code diamond certificate.

**An additional coarse entropy-rate model**

If one separately assumes independent geometric outgoing modes and the same counting convention, define

$$g_2(u) = (u+1)\log_2(u+1) - u\log_2 u, \qquad g_2(0) = 0,$$

and a coarse entropy flux

$$\dot{A}_{\mathrm{cg}}(M) = \sum_\alpha g_\alpha \int_0^\infty \frac{d\omega}{2\pi}\, g_2(n_\alpha(\omega; M)).$$

The geometric-state entropy is *g_2(u)*, obtained by summing its number probabilities. Mode independence makes these entropies additive. This is a coarse state model, not the fine-grained entropy of a variable-code evaporation channel.

Under the same massless scaling,

$$\dot{A}_{\mathrm{cg}}(M) = \frac{\sigma}{M}, \qquad \sigma = \sum_\alpha g_\alpha \int_0^\infty \frac{dx}{2\pi}\, g_2\!\left(\frac{\gamma_\alpha(x)}{e^{8\pi x} - 1}\right).$$

Since $-B = 8\pi\lambda/(M \ln 2)$, its declared ratio is

$$\xi = \frac{\sigma \ln 2}{8\pi\lambda}.$$

Equations and require different state and counting assumptions. If this ratio is not one, using *A_cg* as *log_2 r* changes the fixed-dimensional proxy, by Proposition .

A greybody factor suppresses a selected transmitted occupation. It does not universally specify an ``early-time suppression'' of a Page curve, nor prove typicality of the source code.

**Probability-weighted transfer is a different exact proxy**

Let *C = bigotimes_{j = 1}^K C_j*, with *dim C_j = d_j*, and supply probabilities *q_j ∈ [0, 1]*. On each factor use the flagged transfer isometry

$$|\psi\rangle_{C_j} \longmapsto \sqrt{1-q_j}\,|\psi\rangle_{R_j}|\varnothing\rangle_{B_j} + \sqrt{q_j}\,|\varnothing\rangle_{R_j}|\psi\rangle_{B_j}.$$

The occupied logical sector and vacancy are orthogonal. Let $S \subseteq \{1, \ldots, K\}$ denote the surviving accessible factors and put

$$w_S = \prod_{j \in S}(1-q_j) \prod_{j \notin S} q_j.$$

**Theorem.** For arbitrary input *ρ_C*,

$$S(R) = \sum_{j=1}^{K} h_2(q_j) + \sum_S w_S S(\rho_{C_S}).$$

Its exact whole-product-code recovery error is

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}) = 1 - \prod_{j=1}^{K}\left[1 - q_j(1 - d_j^{-2})\right].$$

These are product-erasure formulas, not Haar Page formulas.

*Proof.* The accessible output is an orthogonal direct sum over flags *S*, with weights *w_S* and states *ρ_C_S*. The flag entropy is the sum of the independent binary entropies, which proves .

Compress every occupied block and append *I_C_j/d_j* on vacancy. The decoded product channel is

$$\mathrm{bigotimes}_j\left[(1-q_j)\mathrm{id}_{C_j} + q_j \mathscr{T}_{I_{C_j}/d_j}\right].$$

Write each replacement as a Weyl average. The identity-conjugation coefficient is *∏_j [1 − q_j(1 − d_j^{−2})]*; separating that term gives the upper bound .

For the lower bound use the product Bell input. On flag *S*, the missing references are independent maximally mixed states and no decoder can attain full Bell overlap larger than *∏_{j ∉ S} d_j^{−2}*. Average over flags. The

resulting overlap bound is the same product coefficient, proving optimality.

This extends the supplied multi-cell calibration in Hawk02-B [CheungHawk02B]. A proposed greybody-to-*q_j* map must still be stated and justified. At *q_j = 0* for every *j*, the code is released, not retained. Entropy turnover alone does not prove that a particular process retains the code; a retaining comparison must be constructed separately.

## Conserved-sector scrambling: a direct-sum Page calculation

A conserved total additive charge need not leave a tensor-product sector. At a cut, suppose the justified sector is

$$\mathscr{H}_{\mathrm{q}} = \mathrm{bigoplus}_{\lambda}\,(R_{\lambda} \otimes B_{\mathrm{q}-\lambda}),$$

embedded in the physical tensor product. Distinct $\lambda$ have orthogonal radiation charge sectors and orthogonal residual charge sectors. Retain only nonzero blocks, and set

$$r_{\lambda} = \dim R_{\lambda}, \quad b_{\lambda} = \dim B_{\mathrm{q}-\lambda}, \quad D_{\lambda} = r_{\lambda} b_{\lambda}, \quad D_{\mathrm{q}} = \sum_{\lambda} D_{\lambda},$$

$$w_{\lambda} = \frac{D_{\lambda}}{D_{\mathrm{q}}}, \qquad m_{\lambda} = \min(r_{\lambda}, b_{\lambda}), \quad n_{\lambda} = \max(r_{\lambda}, b_{\lambda}).$$

**Theorem.** For a Haar pure vector in *𝒽_q*, the mean radiation entropy is

$$\boxed{\mathbb{E}S(R) = \frac{\mathrm{H}_{D_{\mathrm{q}}} - \sum_{\lambda} w_{\lambda} \mathrm{H}_{n_{\lambda}} - \sum_{\lambda} w_{\lambda}(m_{\lambda} - 1)/(2n_{\lambda})}{\ln 2}}.$$

Destructive basis readout resolving each radiation charge sector has mean entropy

$$\boxed{\mathbb{E}H(Y) = \frac{\mathrm{H}_{D_{\mathrm{q}}} - \sum_{\lambda} w_{\lambda} \mathrm{H}_{b_{\lambda}}}{\ln 2}}.$$

The sector geometry and weights cannot be replaced by two aggregate dimensions in the unrestricted Page formula.

*Proof.* Use independent Gaussian coordinates in all sector blocks. Their squared block norms *p_{λ}* have the Dirichlet law with parameters *D_{λ}*. Conditional normalized directions are independent Haar vectors in the individual blocks and independent of the weights.

The residual trace kills cross-charge coherences, so

$$\rho_R = \mathrm{bigoplus}_{\lambda}\, p_{\lambda} \rho_{R_{\lambda}}, \qquad S(\rho_R) = H(p) + \sum_{\lambda} p_{\lambda} S(\rho_{R_{\lambda}}).$$

Dirichlet differentiation gives

$$\mathbb{E}H(p) = \frac{\mathrm{H}_{D_{\mathrm{q}}} - \sum_{\lambda} w_{\lambda} \mathrm{H}_{D_{\lambda}}}{\ln 2}.$$

Add ∑_{λ} *w_{λ}P(r_{λ}, b_{λ})*; the *H_{D_{λ}}* terms cancel, yielding .

Every individual radiation basis outcome in sector $\lambda$ collects *b_{λ}* Gaussian coordinates. Its probability has a Dirichlet parameter *b_{λ}*, repeated *r_{λ}* times. The same moment differentiation gives .

This direct-sum structure is the standard symmetry-resolved random-state setting [SymmetryPage]. Its derivation is included to make the missing tensor-product hypothesis visible.

If the sector probabilities are instead fixed at specified *p_{λ}*, with independent conditional Haar directions, the means become

$$H(p) + \sum_{\lambda} p_{\lambda} \mathrm{P}(r_{\lambda}, b_{\lambda})$$

and

$$H(p) + \sum_{\lambda} p_{\lambda} \mathrm{L}(r_{\lambda}, b_{\lambda}).$$

Externally assigned sector weights define a different ensemble from Haar measure on the whole conserved sector.

**Example.** For *N* qubits with total excitation number one and a cut after *k* qubits, the sector has dimension *N*. For *0*

$< k < N$, its two blocks have dimensions

$$(r_0, b_0) = (1, N - k), \qquad (r_1, b_1) = (k, 1).$$

The coherent reduction has rank at most two and entropy at most one bit. The exact mean is

$$\mathbb{E}S(R_k) = \frac{\operatorname{Harm} N - \frac{N-k}{N}\mathrm{H}_{N-k} - \frac{k}{N}\operatorname{Harm} k}{\ln 2}.$$

The record mean is

$$\mathbb{E}H(Y_{\le k}) = \frac{\operatorname{Harm} N - \frac{N-k}{N}\mathrm{H}_{N-k} - \frac{k}{N}\operatorname{Harm} 1}{\ln 2}.$$

The endpoint coherent entropy is zero, but the curve is not *min (k, N − k)* bits.

More generally, for fixed excitation number *q*,

$$r_\lambda = \frac{k}{\lambda}, \qquad b_\lambda = \frac{N-k}{q-\lambda}, \qquad D_q = \frac{N}{q}.$$

Vandermonde's identity verifies the total sector dimension. A single Haar unitary within that fixed sector and passive cell release give compatible cuts to which Theorem applies. A mass window, non-Abelian angular momentum sector, or interacting energy shell needs its own decomposition; it is not automatically .

Sector projection is also not free postselection. The code must be prepared in the stated sector, or the selective operation and its failure branch must belong to the task.

## Access algebras and independently certified recovery

### Access is not automatically a spatial tensor factor

In a finite physical output space, declare a unital observable algebra $\mathscr{A} \subseteq \mathscr{B}(\mathscr{H}_phys)$. For a tensor factor this can be $\mathscr{B}(R) \otimes I_B$. For a general finite algebra, a decomposition is

$$\mathscr{H}_{\text{phys}} = \operatorname{bigoplus}_\alpha \left(\mathbb{C}^{n_\alpha} \otimes \mathbb{C}^{m_\alpha}\right), \qquad \mathscr{A} = \operatorname{bigoplus}_\alpha \left(M_{n_\alpha} \otimes I_{m_\alpha}\right).$$

Its restricted state can be described by weights $p_\{\alpha\}$ and normalized accessible matrices $\rho_\{\alpha\}$. With the explicitly chosen ordinary matrix trace on each accessible block, one may define

$$S_{\mathscr{A}} = H(p) + \sum_\alpha p_\alpha S(\rho_\alpha).$$

This entropy convention excludes the multiplicity factors $m_\{\alpha\}$; it is not automatically the entropy of an ambient density operator.

Cheung's En05-C compares state restrictions with Heisenberg containment without assuming a spatial complement [CheungEn05C]. A continuum gravitational algebra requires its own representation, trace or entropy prescription, and comparison theorem.

**Proposition.** If a later accessible channel admits the earlier one as CPTP postprocessing, then its unrestricted recovery error cannot increase. No corresponding monotonicity holds for quantum subsystem entropy. Restriction of access alone does not imply a Page curve.

*Proof.* A later decoder can first apply the postprocessing and then an earlier decoder. Quantum entropy can decrease on adding a factor: the first half of a Bell pair has positive entropy, whereas the complete pair is pure. Conversely, a pure product state has zero entropy on every tensor prefix despite the same sequence of increasing access dimensions.

Laddha, Prabhu, Raju, and Shrivastava argue, under their asymptotically flat quantum-gravity assumptions, for information availability near early cuts of null infinity and discuss an entropy behavior unlike the usual Page curve [NullInfinity]. That is an external proposal, not a proof that the present finite proxy is an algebra at infinity. Removing an observable such as the Hamiltonian also requires defining a remaining observable algebra; arbitrary deletion need not leave an algebra closed under multiplication.

### The precise obstruction to duplicate logical algebras

**Proposition.** Let $V: C \to H$ be an isometry. There cannot be two commuting unital star representations $\pi_1, \pi_2: \mathcal{B}(C) \to \mathcal{B}(H)$ such that

$$\pi_1(X)V = VX, \qquad \pi_2(X)V = VX$$

for every $X \in \mathcal{B}(C)$, when $d \geq 2$.

*Proof.* For all $X, Y$,

$$VXY = \pi_1(X)\pi_2(Y)V = \pi_2(Y)\pi_1(X)V = VYX.$$

Injectivity of $V$ would imply $XY = YX$ for every pair of logical matrices, a contradiction.

Two independent commuting matrix algebras can certainly exist. The obstruction concerns their representing the *same* logical input. The partner comparisons in Hawk03-A must retain that distinction [CheungHawk03A]. An island formula is not a theorem of this finite algebraic proposition.

### A deterministic Petz candidate and its certificate

Let $\mathcal{N}: C \to R$ be a supplied finite proxy channel and choose a full-rank code state $\sigma_C$. Set

$$s_R = \widehat{\mathcal{N}}(\sigma_C), \qquad P = \operatorname{supp} s_R,$$

and use the inverse of $s_R$ only on its support. For a fixed code state $\omega_C$, define

$$\mathcal{R}_{\sigma,\widehat{\mathcal{N}}}(Y) = \sigma_C^{1/2}\widehat{\mathcal{N}}\left(s_R^{-1/2}PYPs_R^{-1/2}\right)\sigma_C^{1/2} + \operatorname{Tr}\left((I-P)Y\right)\omega_C.$$

**Lemma.** Equation is CPTP on the entire accessible input space. It is a candidate decoder, not an automatic recovery theorem.

*Proof.* Both terms are CP. The trace of the first is

$$\operatorname{Tr}\left(s_R s_R^{-1/2}PYPs_R^{-1/2}\right) = \operatorname{Tr}(PY).$$

The second contributes $\operatorname{Tr}((I-P)Y)$, proving trace preservation.

Petz and universal recovery channels have important uses in entanglement-wedge reconstruction under additional hypotheses [ChenPetz,CotlerRecovery]. Those geometric conclusions are not imported here.

Let

$$|\Phi_d\rangle = \frac{1}{\sqrt{d}}\sum_i |i\rangle_Q|i\rangle_C, \qquad \Phi_d = |\Phi_d\rangle\langle\Phi_d|, \qquad J_{\mathrm{n}}(\Phi) = (\mathrm{id}_Q \otimes \Phi)(\Phi_d).$$

**Lemma.** For channels with input dimension $d$,

$$\frac{1}{2}\|J_{\mathrm{n}}(\Phi) - J_{\mathrm{n}}(\Psi)\|_1 \operatorname{led}_{\diamond}(\Phi,\Psi) \leq \frac{d}{2}\|J_{\mathrm{n}}(\Phi) - J_{\mathrm{n}}(\Psi)\|_1.$$

*Proof.* The first inequality uses the Bell input. Any pure referenced input can be written $(L \otimes I)|\Phi_d\rangle$, with $\operatorname{Tr}(L^{\dagger}L) = d$ and $\|L\|_{\infty}^2 \leq d$. Multiplying the Choi difference on both sides by $L \otimes I$ bounds its trace norm by $d$ times the Choi trace norm. Convexity handles mixed inputs [Watrous].

**Theorem.** Suppose actual and proxy complete channels are compared under access-preserving identifications with distance at most $\varepsilon$. If

$$\delta_{\mathrm{full}}(\widehat{\mathcal{C}}) \leq \zeta,$$

then

$$\delta_{\mathrm{full}}(\mathcal{C}_{\mathrm{act}}) \leq \min\{1, \zeta + \varepsilon\},$$
$$\varepsilon_{\mathrm{rec}}(\mathcal{N}_{\mathrm{act}}) \leq \min\{a_d, \varepsilon + \sqrt{2\zeta}\}.$$

This is an information-theoretic decoder-existence conclusion.

Alternatively, let a specified proxy decoder $\mathcal{D}$, possibly , have discrepancy

$$r_{\mathrm{dec}} = \frac{1}{2}\left\| J_{\mathrm{n}}(\widehat{\mathscr{D}\mathscr{N}}) - \Phi_d \right\|_1.$$

If its actual implementation has $d_{\diamond}(\mathscr{D}_{imp}, \mathscr{D}) \le e_{imp}$, then that fixed implementation has error at most

$$\min\{1, dr_{\mathrm{dec}} + \varepsilon + e_{\mathrm{imp}}\}.$$

*Proof.* Partial traces contract the complete-channel comparison. The replacement triangle inequality proves leakage. Apply a proxy-optimal decoder to the actual accessible channel and use Lemma . The independent constant decoder gives the $a_d$ cap. For the specified decoder, use Lemma and telescope the channel and implementation differences.

No entropy or greybody statistic was substituted for $\zeta$, $\varepsilon$, or $r_{dec}$. A recovery claim additionally needs the same-model preparation, complete evolution, access identification, and complement certificate emphasized in En07-B and Hawk04-D [CheungEn07B,CheungHawk04D].

Whole-code recovery is also stronger than one-effect recovery. For example, qubit dephasing preserves the basis effect *kb 00* exactly but has whole-code recovery error *1/2*. This is the kind of target separation documented in Oai290-A [CheungOai290A]. Finite streaming workspace obstructions belong to their specified benchmark and resource models, not to every possible decoder [CheungEn02B,CheungEn02C].

### An entropy-history control with opposite recovery

For completeness, append a degenerate inaccessible register $L \simeq C$ to a $b_{*} = 1$ proxy. The returning model scrambles and releases the variable input while fixing $L$. The retaining model stores the variable input in $L$ and runs the same proxy on a fixed dummy input $|0\rangle_C$.

For $|0\rangle_C$, the accessible histories agree exactly for the same unitary. For every fixed pure input, both ensembles have the same Page and record means. At the coherent endpoint, recovery errors are respectively *0* and $a_d$. The retaining complement carries the code isometrically. This proves that the entropy means and the supplied clock do not determine endpoint recovery [CheungHawk01B,Hawk05B]. It does not compare two models under an identical requirement of complete outgoing access and a unique empty residual.

## Cutoffs, recurrence, three ledgers, and analogies

### The cutoff is not an endpoint certificate

A positive $M_{cut}$ specifies the boundary of the supplied clock's validity. It does not establish complement constancy at that boundary, or determine later mass loss, a burst, a remnant, or outgoing quantum access.

Conversely, zero leakage does not force zero energy. The isometry

$$V|\psi\rangle = |\psi\rangle_R|1\rangle_B, \qquad H_B = \varepsilon \mathrm{kb}\, 11, \quad \varepsilon > 0,$$

has zero leakage and positive residual energy. Likewise, a zero-energy residual sector may have a degeneracy that retains the entire code. Energy labels do not count all inaccessible logical degrees of freedom [CheungHawk01B,CheungEn07B].

For a pure, unreferenced complete state, radiation purity is equivalent to purity of its residual marginal. It is not equivalent to channel constancy of that residual on every logical input. The latter is the recovery criterion.

**Proposition.** For a fixed finite-dimensional Hamiltonian $H$, fixed preparation $V_0$, and fixed access partition, the complete channel of $e^{-itH}V_0$ returns arbitrarily close to its initial channel at arbitrarily late times. If the initial accessible channel is replacement, then

$$\limsup_{t \to \infty} \varepsilon_{\mathrm{rec}}(\mathscr{N}_t) = a_d.$$

*Proof.* There are finitely many eigenvalue phases. Simultaneous recurrence on their compact torus gives arbitrarily late times when $e^{-itH}$ is arbitrarily close to identity in operator norm. For isometries, $d_{\diamond}(Ad_V, Ad_W) \le \|V - W\|_{\infty}$, by referenced pure-state comparison and convexity. Partial trace and Lemma give recurrence of the accessible channel and its recovery error. The initial error and universal upper cap both equal $a_d$.

This concerns a fixed closed autonomous finite model [CheungHawk04D]. It does not establish recurrence of an

outgoing continuum radiation interface or an externally terminated cell-release schedule. A regulator limit taken together with late times requires a controlled order of limits.

### Three panels, not three interchangeable numbers

An extrapolation report should keep separate:

$$cM(t),\ P(t),\ \text{declared energy conservation}$$
$$\boxed{\text{heat, work, consumed resources, and cyclic restoration}}$$
$$S(R_t),\ H(Y_{\le t}),\ \delta_{\mathrm{full}}(\mathscr{C}_t),\ \varepsilon_{\mathrm{rec}}(\mathscr{N}_t)$$

Even the information panel contains different quantities. A decreasing mass is not a decreasing fine-grained radiation entropy, and a decreasing entropy is not a recovery certificate.

**Proposition.** A reduced reset channel or partial trace does not determine a unique heat, work, or reusable-memory reset cost.

*Proof.* SWAP a memory with an independent blank qubit. The memory is reset and its referenced input is transferred to the other qubit. For equal gaps $H_M = H_L = \varepsilon kb\ 11$, SWAP conserves their sum and transfers energy $\varepsilon p$, where $p$ is the initial excitation probability. The same reset channel is obtained for every positive $\varepsilon$. Independent fuel, bath, or actuator processes may also change the implementation ledger without changing that channel. The consumed blank and fuel must be restored before a cyclic cost is claimed.

This underdetermination is the En07-A ledger distinction [CheungEn07A]. A thermal erasure bound belongs to an additionally specified memory–reservoir setup, not to $dropBit \circ Enter = Tr$ [ReebWolf].

### Processor, film, album, and library analogies

**Processor and coherent film.** A processor scrambles a pure prepared code and releases unequal quantum cells. The receiver keeps those cells coherently. The Haar hypothesis supplies the finite Page mean. Replacing the equal-cell clock by a compatible release schedule changes the horizontal axis, not the output type.

**Click album.** Installing Enter on every emitted cell produces an append-only classical record. Its exact proxy entropy mean is , not . Later clicks may improve answers to classical questions, but they cannot restore an arbitrary unknown quantum state from that record. Postprocessing an entanglement-breaking channel remains entanglement breaking [HorodeckiShorRuskai,CheungEm03F].

**Torn strip.** For a classical message $L$, more photographs can decrease $H(L \mid Y_{\le k})$ while the full album entropy increases. For a quantum strip, photographs retain outcomes rather than coherent fragments. Successful quantum recovery needs coherent fragments or another explicitly accessible quantum carrier.

**Library cards.** Photocopies can reconstruct known-alphabet classical text. A classical index can also select a decoder acting on a genuine quantum output. It is not itself the unknown quantum book. Emptying a named drawer does not erase unreturned purifiers or measurement environments.

A coherent packet decoder in a declared source-field model is a useful comparison, not a local Schwarzschild receiver [CheungEm03G]. The analogy's usefulness is preserved by stating that missing identification.

## A disciplined extrapolation programme

1.12 toprule Possibility What must additionally be supplied

midrule Clocked fixed-capacity proxy Integer compatible dimensions, one nested history, and ; a power law alone is insufficient.

Greybody-weighted capacities An explicit occupation-to-capacity rule, cutoff and tail, and a typicality hypothesis for that rule.

Two return-type curves One complete process with two declared receiver/readout continuations; publish both entropy definitions.

Restricted exterior access A genuine observable algebra or factor, its state restriction, and an entropy convention; no emergence follows from restriction.

Positive-mass wall The validity interval and the crossover-before-cutoff test; do not attach an unproved zero-entropy endpoint.

Petz or island decoder comparison A deterministic task channel, correct supports, complete comparison allowance, and executable implementation budget.

Conserved-sector scrambling The actual sector decomposition and its ensemble weights; use , not an unrestricted tensor formula.

Three-ledger report Energy, reset resources, entropy, and recovery estimates must retain their different hypotheses and units.

Gravitational conclusion One microscopic preparation, dynamics, access dictionary, receiver, typicality theorem, and endpoint comparison.

bottomrule

The following are specifically not imported:

• a static area-law or catalogue record as a temporal channel history [CheungOai265A];

• a prepared entropy sequence as Hawking emission [CheungEn02D];

• an island or replica-wormhole reference as a complete evaporation isometry [IslandReview];

• an autonomous finite source-field Hamiltonian as a gravitational collapse Hamiltonian [CheungHawk02C];

• any soft, lost, purifier, or other admitted output factor removed from complete accounting merely by a final mass label.

If an actual theory includes such factors, the complete interface must account for them or prove why they are irrelevant to the declared finite code. This paper does not assume their existence, absence, or specific dynamics.

## Hawk05-C release theorem

**Theorem.** Within the stated finite interface discipline:

• nonuniform nested tensor release has the exact coherent and record means ;

• its fixed-dimensional capacities obey ; a coarse entropy-production ratio $\xi \neq 1$ cannot be inserted as a mere clock change;

• the compatible inverse-square example has the explicit rounded two-curve sketch –, while the general supplied capacity envelope has the crossover and cutoff conditions of Theorem ;

• additive-charge-sector Haar scrambling has the exact direct-sum means –;

• physical two-curve transfer is conditional on the separate capacity, ensemble, channel, and detector allowances in –;

• whole-code recovery is a separate complete-complement or decoded-channel certificate, with bounds and ;

• every record-only endpoint has the permanent error lower bound $1 - 1/d$, regardless of the clock or capacity plot.

These results establish finite extrapolation possibilities and their compatibility tests. They do not construct a gravitational Page curve, an island, spacetime emergence, or the inherited characteristic quantum coefficient.

*Proof.* Items (i)–(iv) are Theorems , , , , and , with Proposition . Item (v) is Theorem . Item (vi) is Theorem and the exact criterion of Theorem . Item (vii) is Proposition . No physical comparison theorem supplying the excluded identifications is among the constructed objects.

**Final conclusion.** The clean next calculation is a compatible capacity clock with two declared return types and explicit, unfilled physical comparison slots. The coherent proxy may turn over; the unedited classical album cannot. A crossover of capacities, a maximum of a mean entropy, a decrease of complete leakage, and an executable recovery threshold remain four different events until a same-model theorem relates them.

appendix

## The Page–Sen calculation used in the proxy

We record the finite calculation to make the imported random-state input explicit. The result is due to Page and Sen [PageAverage,Sen]; no priority is claimed.

Let $m \le n$, $D = mn$, and let $G$ be an $m \times n$ standard complex Gaussian matrix. Put

$$W = GG^{\dagger}, \qquad T = \operatorname{Tr} W, \qquad \rho = W/T.$$

Gaussian radius and direction are independent, $T$ is Gamma with shape $D$, and $\rho$ is the Haar reduction. Hence, in natural entropy units,

$$\mathbb{E}S_{\mathrm{nat}}(\rho) = \psi_0(D+1) - D^{-1}\mathbb{E}\operatorname{Tr}(W\ln W),$$

because $Tr\ (Wln\ W) = Tln\ T + TTr\ (\rho ln\ \rho)$ and $\mathbb{E}(Tln\ T) = D\psi_0(D + 1)$.

Write $\alpha = n - m$. The singular-value Jacobian gives the eigenvalue density proportional to

$$e^{-\sum_i x_i}\prod_i x_i^{\alpha}\prod_{i<j}(x_i - x_j)^2, \qquad x_i > 0.$$

Orthogonalizing the Vandermonde with generalized Laguerre polynomials gives the one-point density

$$R_1(x) = e^{-x}x^{\alpha}\sum_{j=0}^{m-1}\frac{[L_j^{\alpha}(x)]^2}{h_j}, \qquad h_j = \frac{\Gamma(j+\alpha+1)}{j!}.$$

These statements follow by the singular-value change of variables and expansion of the squared orthogonal-polynomial determinant.

Rodrigues's formula and integration by parts give the norms and orthogonality. The identities

$$xL_j^{\alpha} = (2j+\alpha+1)L_j^{\alpha} - (j+1)L_{j+1}^{\alpha} - (j+\alpha)L_{j-1}^{\alpha},$$

$$\partial_{\alpha}L_j^{\alpha} = \sum_{\ell=0}^{j-1}\frac{L_{\ell}^{\alpha}}{j-\ell}$$

follow respectively from coefficient comparison and differentiation of the generating function

$$\sum_{j\ge 0}L_j^{\alpha}(x)t^j = (1-t)^{-\alpha-1}\exp\left(-\frac{xt}{1-t}\right).$$

Consequently

$$\frac{1}{h_j}\int_0^{\infty}x^{\alpha+1}e^{-x}\ln x[L_j^{\alpha}(x)]^2\,dx = (2j+\alpha+1)\psi_0(j+\alpha+1) + 1 + 2j.$$

Indeed, differentiate

$$\int_0^{\infty}x^{\alpha+1}e^{-x}[L_j^{\alpha}(x)]^2\,dx = (2j+\alpha+1)h_j;$$

the polynomial derivative cross term is $-jh_j$. Differentiation under the integral is justified locally for $\alpha > -1$ by the exponential weight and integrability at zero.

Summing the displayed logarithmic integrals gives

$$\mathbb{E}\operatorname{Tr}(W\ln W) = mn\psi_0(n+1) + \frac{m(m-1)}{2}.$$

For example, substitute $\psi_0(r) = \psi_0(n + 1) - \sum_\{\ell = r\}\^n\ \ell\^\{-1\}$ and use

$$\sum_{r=\alpha+1}^{\ell}(2r-\alpha-1) = \ell(\ell-\alpha)$$

to evaluate the finite double sum. Equation becomes

$$\mathbb{E}S_{\mathrm{nat}}(\rho) = \psi_0(mn+1) - \psi_0(n+1) - \frac{m-1}{2n}.$$

Using $\psi_0(\ell + 1) = H_\{\ell\} - \gamma_E$ and dividing by $ln\ 2$ proves Lemma , including $m = 1$.

## The independent characteristic-map boundary

The supplied XIV-I, XIV-J, and XV-B passages concern a different distributional programme [CheungXIVI,CheungXIVJ,CheungXVB]. The finite capacity and entropy results above do not act on its six-branch characteristic target.

For a surjective reconstruction map *Rec:* $\mathcal{P} \to \mathcal{K}$, a presentation-level linear map $M$ descends uniquely exactly when

$$\ker \mathrm{Rec} \subseteq \ker \widetilde{M}.$$

This follows directly because equal reconstructions differ by a reconstruction-null presentation. The identity

$$q\delta'(q) + \delta(q) = 0$$

likewise prevents a representative-dependent highest degree from automatically defining an intrinsic reconstructed symbol: pairing *qδ^{'}* with a test function gives minus its value at zero.

These compatibility facts do not construct the full joint characteristic map. According to the supplied versions, the named internal amplitudes are constructed at I2, while the full map remains unresolved at I3/J2 and in the graph-specific XV-B continuation. The required section, complete remainder, transverse estimates, contact compatibility, output wavefront theorem, and normalized quantum admission are not supplied here.

The standing supplied boundaries remain unchanged: *O. 1 = texttt NOT_CLOSED*; the downstream quantum-admission exit is not opened; XVI-N's universal obligation is not discharged; and no inherited characteristic quantum coefficient is admitted. The conditional massless obstruction in the supplied XVI-N instance is not contradicted or enlarged [CheungXVIN]. These are scoped provenance statements, not universal nonexistence theorems.

## Declarations

**Attribution.** The Page average, finite entropy-continuity theorem, exact correction criterion, entanglement-breaking structure, and information–disturbance theorem are established results cited at their use. The direct-sum sector calculation is consistent with the symmetry-resolved random-state literature. The contribution is their assembly into a corrected extrapolation framework, the capacity compatibility test, the two-return-type clocked sketch, and explicit separation of the required allowances.

**Source provenance.** Cheung's internal manuscripts are cited by the supplied identifiers and versions. Documentary comparison is not independent verification of the entire corpus and is not an identification of their models. The supplied Hawk05-A and Hawk05-B author fields are empty; authorship of those files is not inferred. Cheung's journal article is distinguished from the internal manuscripts.

**Physical scope.** No gravitational microscopic Hamiltonian, collapse channel, backreaction law, local coherent receiver, island surface, replica-wormhole calculation, or spacetime-emergence theorem is constructed. The supplied clock and black-hole entropy law do not remove those obligations.

**Build and verification.** This is a standalone source written for pdflatex. No successful compilation, proof-assistant verification, numerical execution, independent referee approval, or journal acceptance of Hawk05-C is asserted.

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